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# The *Proslogion*: Philosophy and Logic

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## 1. Introduction[1]

The *Proslogion*, written by Anselm of Aosta, is certainly one of the most original works in the history of philosophical and theological thought, so much so that it continues to arouse heated debates and innovative research.[2] Currently, the vast criticism tends to be focused on the famous chapter 2,[3] in which the existence of *id quo maius cogitari non possit* (or *id quo maius cogitari nequit*: "something than which a greater is not conceivable" abbreviated as IQM)[4] is demonstrated, but in

[1] This essay is largely indebted to conversations and correspondence with Francesco Berto and David Cerny whom we thank and of course excuse from any errors contained in this document. We also thank the reviewers of this article for their relevant comments and suggestions.

[2] On the history of ontological arguments, see R.G. Tomossi, *Prove logiche dell'esistenza di Dio: da Anselmo a Kurt Gödel*, Milano 2005; F. Tomatis, *L'argomento ontologico: l'esistenza di Dio da Anselmo a Schelling*, Milano 2010; G. Piazza, *Il nome di Dio*, Bologna 2000; G. Oppy, *Ontological Arguments*, in: *The Stanford Encyclopedia of Philosophy* (Spring 2019 Edition), ed. E.N. Zalta, URL: https://plato.stanford.edu/archives/win2021/entries/ontological-arguments/.

[3] An exception that is worth noting is the important article of J. Archambault *Monotonic and Non-Monotonic Embeddings of Anselm's Proof*, "Logica Universalis" 2017, Vol. 11, pp. 121–138, where the entire structure of the *Proslogion* is analyzed using non-monotonic logics.

[4] Anselm, *Proslogion*, in: Anselm, *Monologion and Proslogion: With the Replies of Gaunilo and Anselm*, ed. T. Williams, Indianapolis 1995, ch. 2, p. 100.

fact the whole of Anselm's work has a broader and unitary structure that can be divided into three parts:[5]

1° Beginning: invocation and prayer (ch. 1)

2° Ascent:

I. Chapter 2: IQM *exists in reality* (principle of existence)

II. Chapters 3–13: IQM *is the Best of all, than which nothing better is conceivable* (*Summum Omnium, quo nihil Melius cogitari potest*; principle of excellence)

III. Chapters 14–15: IQM is not conceivable (*Quiddam Maius quam cogitari possit*; principle of transcendence)

IV. Chapters 16–23: other divine attributes which are demonstrated or clarified in the light of the three preceding principles

3° Ending: the fullness of joy (ch. 24–26)

In this article, which has to be considered only an interpretative proposal that could be developed in future studies, we wish to highlight this articulation through two methods of analysis: informal [2] and formalized [3].

The theses we will try to demonstrate are the following:

1) The line of demonstration (also known as *unum argumentum*[6]) contained in this work is only valid for IQM and not for similar entities.
2) The work has an overall unity and an ascending trend [4].

Parts 2–3 of this article have the same structure and numbering of the paragraphs, because they express the same contents, albeit through two different types of language (informal vs formalized), for which they are "readable" independently, depending on the training of the reader. However, it is recommended that the readers acquaint themselves with both parts in order to adequately grasp the depth and rigour of Anselm's work: for this reason, we have tried to make the formal part accessible even to non-specialists through intuitive explanations and concrete examples.

[5] See I. Sciuto, *Introduzione*, in: Anselmo, *Proslogion*, ed. I. Sciuto, Milano 1996, pp. 5–76. In this paper we will use the English version edited by T. Williams (Anselm, *Monologion and Proslogion: With the Replies of Gaunilo and Anselm*, ed. T. Williams, Indianapolis 1995) of which we have translated "cogitare" as "conceive" (rather than "think"), and "in intellectu" as "in the intellect" (rather than "in understanding"); the Latin fragments are quoted from Anselmo, *Proslogion*, ed. I. Sciuto, Milano 1996.

[6] Anselm, *Proslogion*, op. cit., Proemium.

## 2. Informal Analysis of the *Proslogion*

### 2.0. Premise[7]

Before proceeding with the analysis, some observations on the basic "concepts" of the *Proslogion* are appropriate:

a) "Something than which a greater is not conceivable" indicates something understandable but which, at least at the beginning of the path, has no determined positive content;
b) In this sense, Anselm distinguishes between existence within the intellect (for example, the image of a painting that you want to paint; the definition of something, for instance an island) and existence in reality (the actual painting or island).

> For it is one thing for an object to exist in the intellect and quite another to understand that the object exists [in reality]. When a painter, for example, conceives in advance what he is going to paint, he has it in his intellect, but he does not yet understand that it exists [in reality], since he has not yet painted it. But, once he has painted it, he has it both in his intellect and understands that it exists [in reality] because he has now painted it.[8]

c) Another fundamental distinction is between the ability to conceive ("cogitare") and to understand ("intelligere")[9]: according to Anselm we can only understand something true (for example, if Socrates exists, we can understand that Socrates exists but we cannot understand that Socrates does not exist), while we can conceive something different from what we have understood (for instance, if we have understood that Socrates exists, we can conceive that Plato does not exist); however, Anselm denies that we can conceive a contradiction (for example, "Socrates exists and Socrates does not exist"):

7 This section is based on C.A. Testi, *Unicità e unità dell'Unum Argumentum di Anselmo d'Aosta*, "Aquinas" 2021, Vol. 2, pp. 473–491.

8 Anselm, *Proslogion*, op. cit., ch. 2, p. 100. "Aliud enim est rem esse in intellectu, aliud intendere rem esse. Nam cum pictor praecogitat quae facturus est, habet quidem in intellectu, sed nondum intelligit esse quod nondum fecit. Cum vero iam pinxit, et habet in intellectu et intelligit esse quod iam fecit."

9 J. Barnes correctly distinguishes "esse in intellectu," "intelligere" and "cogitare" (*The Ontological Argument*, London–Basingstoke 1972, pp. 9–11).

> For even if nothing that actually exists can be understood not to exist [we cannot understand something different from reality, but only understand what really is], everything can be conceived to not exist [we can conceive something different from this reality], except for that which exists supremely [see 2.2, 3.2]. And yet we cannot conceive of something as not existing even while we know that it exists, since we cannot conceive of it as existing and not existing at the same time.[10]

Conceiving also differs from affirming or simply hypothesizing (see 3.0.6 RCo1–2).

We now turn to illustrating the ascent (2°) in its three central speculative steps I–II–III.

## 2.1. IQM Exists in Reality (Ch. 2)

### 2.1.1. *Unum Argumentum*: Logical Structure

Here is the famous passage from chapter 2, which we have divided into six passages:

> (1) So even the fool must admit that something than which nothing greater can be conceived exists at least in his intellect, since he understands this when he hears it, and whatever is understood exists in the intellect. And surely that than which a greater cannot be conceived cannot exist only in the intellect. (2) For if it exists only in the intellect, (3) it can be conceived to exist in reality as well, (4) which is greater. (5) So if that than which a greater cannot be conceived exists only in the intellect, then the very thing than which a greater cannot be conceived is something than which a greater can be conceived. But that is clearly impossible. (6) Therefore, there is no doubt that something than which a greater cannot be conceived exists both in the intellect and in reality.[11]

[10] Anselm, *Reply to Gaunilo*, in: Anselm, *Monologion and Proslogion: With the Replies of Gaunilo and Anselm*, ed. T. Williams, Indianapolis 1995, p. 133. “Nam et si nulla quae sunt possit intelligi non esse, omnia tamen possit cogitari non esse, prater id quo summe est. […] Et non possumus concepire non esse, quamdiu scimus esse, quia non possumus concepire esse simul et non esse.”

[11] Anselm, *Proslogion*, op. cit., ch. 2, p. 100. “1) Convincitur ergo etiam insipiens esse vel in intellectu aliquid quo nihil maius cogitari potest, quia hoc cum audit intelligit, et quidquid intelligitur in intellectu est. Et certe id quo maius cogitari nequit, non potest esse in solo intellectu. 2) Si enim vel in solo intellectu est, 3) potest cogitari esse et in re, 4) quod maius est. 5) Si ergo id quo maius cogitari non potest, est in solo intellectu: id ipsum quo maius cogitari non potest, est

The argument, therefore, has the following structure:
1) It refers IQM to the intellect.
2) It assumes that this does not have a certain property.
3) It conceives something that *has* said property instead.
4) In the light of a hierarchy principle, it can be deduced that the latter is greater than IQM.
5) A contradiction follows.
6) It negates hypothesis 2, which caused contradiction 5.

The proof of the existence of IQM can therefore be rewritten as follows:
1) If someone says "something than which a greater is not conceivable," he understands what he is saying; therefore, something than which a greater is not conceivable exists at least in one's intellect.
2) Let's assume that this *does not exist in reality* (= exists *only* in the intellect).
3) If so, it would be conceivable that something than which a greater is not conceivable exists in reality.
4) But then, since what exists in reality is greater than what does not exist in reality, something greater than what was indicated in 1) as something than which a greater is not conceivable, *is conceivable*.
5) It would follow that something than which a greater *is not* conceivable (1) is something than which a greater *is* conceivable (4).
6) However, 5) is contradictory; therefore, hypothesis 2, from which this contradiction derives, must be denied, and therefore something than which a greater is not conceivable exists in reality.

### 2.1.2. *Unum Argumentum* and Specific Entities

It is now a question of understanding why this proof applies only to IQM and not to specific entities. Consider, for example, the concept of "the island than which a greater island is not conceivable," which takes up Gaunilo's objection based on the idea of a "fabulous island."[12] Now, this notion, apparently superimposable on IQM, is actually much more complex because:

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quo maius cogitari potest. Sed certe hoc esse non potest. 6) Existit ergo procul dubio aliquid quo maius cogitari non valet, et in intellectu et in re."

12 According to Gaunilo, with Anselm's argument it is also possible to demonstrate an "island, more excellent than all others on earth" (Gaunilo, *Answer on Behalf of the Fool*, in: Anselm, *Monologion and Proslogion: With the Replies of Gaunilo and Anselm*, ed. T. Williams, Indianapolis 1995, p. 125: "insulam illam terris omnibus praestantiorem vere esse alicubi in re"). Here, in ad-

- it thematizes an entity of a certain type (an island), which was not the case for IQM, whose nature and properties were not assumed;
- contains three conditions whose non-conceivability is declared, namely, that:
  a1 – this is an island;
  a2 – and something is an island;
  a3 – which is greater than this.

Let's see what happens to the proof using the same demonstration strategy:

1) If someone says "this island than which a greater island is not conceivable" (or "it is not conceivable that this is an island and something is greater than this"), he understands what he is saying; therefore, this island than which a greater island is not conceivable exists at least in one's intellect.
2) Let's assume that this does not exist in reality.
3) If so, it would be conceivable that the island than which a greater island is not conceivable and such that it exists in reality.
4) But then, since what exists in reality is greater than what does not exist in reality, it is conceivable that that (conceivable in 3) is greater than this (in 1).

But 1) ("it is not conceivable that this is an island and something is an island greater than this") and 4) ("it is conceivable that that is greater than this") do not contradict each other, one not negating the other, while in *unum argumentum* 4) was the exact negation of 1). From 4) it follows at most that this island is not IQM, given that one can conceive something greater. Nor can "the island than which a greater island is not conceivable" enjoy (as IQM: see 2.2) the property of "being inconceivable as non-existent in reality" because, as a given limited entity, it is always conceivable as non-existing in reality: this is how Anselm replies to Gaunilo.[13]

### 2.1.3. *Unum Argumentum* and the Greatest

Similar considerations can also be made for demonstrations centred on the concept of "greatest": in fact, even admitting that "something greater than everything else" exists in our intellect, if we assume that this does not exist in reality, it does

dition to the error of applying the argument to specific entities, he also uses the concept of "best" ("more excellent than all") and not that of "something than which a greater is not conceivable," as Anselm then points out (Anselm, *Reply to Gaunilo*, op. cit., ch. 3, p. 134 ff.).

13 Anselm, *Reply to Gaunilo*, op. cit., ch. 3, p. 132.

not follow that it exists in reality, but only that an existing greater is conceivable, that is conceivable as greater than the not existing greatest:

> For what if someone were to say that something exists that is greater than everything else that exists, and yet that this very thing can be conceived not to exist, and that something greater than it can be conceived, although that greater thing does not actually exist? […] In this case we would need another premise, besides the mere fact that this being is said to be "greater than everything else."[14]

Anselm in fact adds that, although the existence of the greatest cannot be demonstrated, it is possible to demonstrate that IQM, which exists in reality, must also be the greatest, for the same reasons as above: if IQM were not the greatest, since the greatest is greater than the non-greatest, it would be possible to conceive a greater than that which a greater is not conceivable, which is contradictory (see 3.1.3):

> For that than which a greater cannot be conceived cannot be understood as anything other than the one thing that is greater than everything else. Therefore, just as that than which a greater cannot be conceived is understood and exists in the intellect, and therefore is affirmed to exist in actual fact, even so that which is said to be greater than everything else is with necessity inferred to be understood, to exist in the intellect, and consequently to exist in reality.[15]

Also using the concept of "conceivable as greater than everything," similar limitations are encountered: in fact, as for the island, if the conceivable as greater than everything else does not exist, it follows only that something greater than it will be conceivable, but this is quite different from proving that it exists in reality (see 3.1.3).

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[14] Ibid., ch. I.5, p. 135. "Quid enim si quis dicat esse aliquid maius omnibus quae sunt, et idipsum tamen posse cogitari non esse, et aliquid maius eo etiam si non sit, possit tamen cogitari? […] Illud tamen indigiet alio argomento quam hoc quod dicitur 'omnibus maius.'"

[15] Ibid. "Nullatenus enim postest intelligi 'quo maius cogitari non possit' nisi id quod solum omnibus est maius. Sicut ergo 'quo maius cogitari nequit' intelligitur et est in intellectu, st ideo esse in rei veritate asseritur: sic quod maius dicitur omnibus, intelligi et esse in intellectu, et idcirco re ipsa esse ex necessitate concluditur."

#### 2.1.4. *Unum Argumentum* and the Perfect Entity (*Ens Perfectissimum*)

The proof is not valid even if, instead of IQM, we use the idea of the perfect entity (see Descartes and most of the moderns).[16] In fact:

- If we intuitively define a perfect entity as a being possessing all perfections, surely it can be deduced that, *if* something can be called the perfect entity, then this must exist in reality, since existence in reality is a perfection: however, as is clear from the emphasized "if," we are far from proving that the perfect entity exists in reality.
- Even if, as an analogy with IQM, the concept of a perfect entity is extended to its conceivability by stating that "it is conceivable that something is a perfect entity," if we assume that this does not exist in reality, we can at best deduce that "it is conceivable that this is not a perfect entity," which is not a negation of the starting point (that would instead be "it is *not* conceivable that this is a perfect entity"; 3.1.2 Theor 1.4).

### 2.2. IQM Is the Best of All, Than Which Nothing Better Is Conceivable (*Summum*, *Melius*: Ch. 3–14)

However, the existence of IQM is only the first of a series of steps that gradually attest to other positive properties enjoyed by IQM. These steps are based on the

[16] Descartes said: "But when I concentrate more carefully, it is quite evident that existence can no more be separated from the essence of God than the fact that its three angles equal two right angles can be separated from the essence of a triangle, or than *the idea* of a mountain can be separated from *the idea* of a valley. Hence it is just as much of a contradiction to think of God (that is, a *supremely perfect being*) lacking existence (that is, lacking a perfection), as it is to think of a mountain without a valley" (Descartes, *Meditations on First Philosophy*, in: Descartes, *Meditations on First Philosophy: With Selections from the Objections and Replies*, eds. J. Cottingham, B. Williams, Cambridge 2017, V Meditation, p. 52, italics added). The passage clearly shows how the French philosopher manages only to demonstrate that *the idea* of "perfect" implies *the idea* of simplicity and identity between essence and existence, and therefore the *idea* of existence; but it does not prove that the perfect exists. Leibniz observed as much and criticized how Descartes had not previously demonstrated that the idea of a perfect being is possible (e.g., not contradictory; see G. Piazza, *Il nome di Dio*, op. cit., p. 102 ff.). Almost all modern philosophers (Spinoza, Malebranche, Wolf, Baumgarten, Kant, Hegel) work from the idea of a perfect being; see also F. Tomatis, *L'argomento ontologico…*, op. cit.; J.H. Sobel, *Logic and Theism: Arguments For and Against Beliefs in God*, Cambridge 2003, p. 48 ff.). On the logical-mathematical side, Gödel also formalizes the ontological proof, drawing inspiration from Leibniz (see K. Gödel, *Ontological Proof*, in: K. Gödel, *Collected Works. III: Unpublished Essays and Lectures*, eds. S. Feferman, J. Dawson, S. Kleene, G. Moore, R. Solovay, J. van Heijenoort, Oxford 1995, pp. 403–404; G. Lolli, P. Odifreddi, *Kurt Gödel: la prova matematica dell'esistenza di Dio*, Milano 2006).

same demonstration line used above, with the difference that, in step 4, the principle contained therein is modified. Basically, if in the phrase "since what *exists in reality* is greater than what does not exist in reality" we replace "exists in reality" with:

2) is inconceivable as non-existent in reality (ch. 4);
3) is the creator of all things from nothing (ch. 5);
4) is all that is better to be than not to be (ch. 5);
5) is most perceptive (ch. 6);
6) is omnipotent (ch. 7);
7) is merciful (ch. 8);
8) is impassable (ch. 8);
9) is supremely just (ch. 9–11);
10) is supremely good (ch. 9–11);
11) is blessed (ch. 11);
12) is per se (ch. 12);
13) is limitless (ch. 13);
14) is omnipresent (ch. 13);
15) is eternal (ch. 13).

It will be shown that IQM exists in reality and also has all these fourteen perfections so that IQM can also be understood as the Best of all, than which nothing better is conceivable ("summum omnium, quo nihil melius cogitari potest").[17]

Of particular interest is the second demonstrated positive property of IQM, as it concerns its conceivability. In fact, in principle, it is conceivable that an IQM does not exist in reality; Anselm, however, here rejects this possibility by showing that "IQM is inconceivable as non-existent in reality," along with the line of argument set out in 2.1.1:

> (1) This [being] exists so truly that (2) it cannot even be conceived not to exist [in reality]. (3) For it is possible to conceive that something exists [in reality] that cannot be conceived not to exist [in reality], and such a being is greater than one that can be conceived not to exist [in reality]. (4) Therefore, if that than which a greater cannot be conceived can be conceived not to exist [in reality], then that than which a greater cannot be conceived is not that than which a greater cannot be conceived; (5) then this is a contradiction. (6) So

[17] Anselm, *Proslogion*, op. cit., ch. 14.

> that than which a greater cannot even be conceived exists so truly that it cannot be conceived not to exist [in reality].[18]

### 2.3. IQM Is Not Conceivable (*Maius Quam Cogitari Possit*: Ch. 15)

In chapter 15 we reach the pinnacle of the *Proslogion* since we demonstrate that IQM, in addition to existing in reality and being the Best, is also greater than any different conceivable. Sciuto calls this the "principle of transcendence"[19] because it can even be read as a statement that IQM is not conceivable:

> Therefore, Lord, you are not merely that than which a greater cannot be conceived; you are something greater than can be conceived. For since it is possible to conceive that such a being exists, then if you are not that being, it is possible to conceive something greater than you. But that is impossible.[20]

As can be seen from this short passage, the usual demonstration strategy is used, in which we replace "exists in reality" with "is greater than any different conceivable":

1) If someone says "something than which a greater is not conceivable," such a person understands what he is saying, so that something than which a greater is not conceivable exists at least in one's intellect.
2) Let's assume that this is not greater than any different conceivable.
3) If so, it would be in fact possible to conceive something than which a greater is not conceivable, which is also greater than any different conceivable.
4) But then, since what is greater than any different conceivable is greater than what is not greater than any different conceivable, one conceives something greater than what is indicated in 1) as something than which a greater is not conceivable.

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[18] Ibid., ch. 3, pp. 100–101: "Quo utique sic vere est, ut nec cogitari possit non esse. Nam potest cogitari esse aliquid, quod non possit cogitari non esse; quod maius est quam quod non esse cogitari potest. Quare si id quo maius nequit cogitari, potest cogitari non esse: id ipsum quo maius cogitari nequit, non est id quo maius cogitari nequit; quod convenire non potest. Sic ergo vere est aliquid quo maius cogitari non potest, ut nec cogitari possit non esse."

[19] I. Sciuto, *Introduzione*, op. cit., p. 47 ff.

[20] Anselm, *Proslogion*, op. cit., ch. 15, p. 109. "Ergo domine, non solum es quo maius cogitari nequit, sed es quiddam maius quam cogitari possit. Quoniam namque valet cogitari esse aliquid huiusmodi: si tu non es hoc ipsum, potest cogitari aliquid maius te; quod fieri nequit."

5) It would follow that something than which a greater is not conceivable (1) is something than which a greater *is* conceivable (4).
6) However, 5 is contradictory; therefore, hypothesis 2, from which this contradiction derives, must be denied, meaning that something than which a greater is not conceivable *is greater than any different conceivable*.

This reading then gives in to an almost mystical discourse because, if one interprets the very short text of chapter 15 in a certain way, it follows that IQM is not conceivable. If, in fact, it were conceivable then it would be possible to conceive something so great that it is not conceivable, and this would be greater than that of which nothing greater is conceivable, which is a contradiction (see 3.3). This extraordinary result is, in our opinion, also confirmed in the rest of the *Proslogion*: it will be shown, for example, that IQM is the supreme light (principle of perfection), but also inaccessible (principle of transcendence):

> Lord, this is the inaccessible light in which you dwell. […] My intellect cannot see that light my intellect does not grasp it, and the eye of my soul cannot bear to look into it for long. It is dazzled by its splendor, vanquished by its fullness, overwhelmed by its vastness, perplexed by its extent. O supreme and inaccessible light.[21]

Similar proceedings occur in the subsequent chapters with the attributes of: harmony, perfume, taste, sweetness, beauty (ch. 17), simplicity (ch. 18), omnipresence (ch. 19), antecedence and superiority above all things (ch. 20), eternity (ch. 21), He who is (ch. 22), tri-unity (ch. 23).

## 3. Logical Analysis of the *Proslogion*

### 3.0. On the Formal Language Adopted

#### 3.0.1. Premise: Basic Concepts

Here we will propose an analysis of the *unum argumentum* with a formal language which seeks to be:

[21] Ibid., ch. 16, p. 110, italics added. "Vere, domine, haec est lux inaccessibilis, in qua habitas. […] Non potest intellectus meus ad illam. Nimis fulget, non capit illam, nec suffert oculus animae meae diu intendere in illam. Reverberatur fulgore, vincitur amplitudine, obruitur immensitate, confunditur capacitate. O *summa et inaccessibilis* lux."

- as close as possible to the text of the *Proslogion*;
- as simple as possible, precisely because the *unum argumentum* had to be understandable to everyone.

For these reasons we will avoid using modal logic and we will leave the more "technical" logical comments in the footnotes and in the appendices.

To develop this analysis the same contents as those showed in part 2 will be expressed in a more formal language. Thus, more symbols will be used, and we will take for granted the syntax and semantics of the classical propositional calculations and predicative calculations (PC).

It is important to emphasize that the aim of this article is to express more precisely the content of the *Proslogion* and Anselm's ideas. For this reason, we do not consider it necessary to carry out a proof of the consistency of the theory, which could be attempted in a future article of an exclusively formal character; however, in Appendix B we develop some theorems regarding the inconsistency of another version of the theory with an additional axiom and the use of quantifiers in our theory.

Let's start with our analytical interpretation of the "notion" of "id quo maius cogitari non possit":

a) "Id": this refers to something, and for this we will use "$\exists x$" ("for something") and "$x$" ("something") (see 3.0.2);
b) "Quo": "of which," indicates a relationship that "id" must have with something else – this something (the "of which" of "$x$") can be expressed by referring to the variable "$y$" (which stands for any noun constant: "$\forall y$")
c) "Maius": we will read this notion in the sense of a strict great order ("$>$": see RCo1–2);
d) "Cogitari": this decisive verb concerns the possibility of conceiving situations in which a subject does or does not have a certain predicate so, logically, it takes as predicates any formula with free variables and possibly some bound variables, and including propositions, considered as nullary predicates, as arguments. Conceivability is governed by two rules (RCo 1–2), which forbid conceiving what leads to a contradiction, but allow for conceiving that that we have not understood (see 2.0.c for the difference between "to conceive" and "to understand").
e) "Non possit": we interpret this as a simple negation, without resorting to the notion of the possibility of modal logic, in light of the equivalent for-

mulation of "id quo maius cogitari nequit" in which "possit" does not appear.

During formalization we will also be able to thematize two notions of existence:

f) *Existence in the intellect*: something exists in the intellect when it is accepted as the principle of a theory or is recalled as a line within a demonstration (see Pr IQM and Theor 1.1, line 1);
g) *Existence in reality*: we will use the predicate "*ex*(…)" which applies precisely to what exists in reality, and therefore does not exist in the intellect alone.

### 3.0.2. Primitive Symbols and Their Interpretation

We will use the usual symbols for:

- {, [, (, ), ], }: usual brackets.
- *x*, *y*, *z*: noun variables.
- *a*, *b*, *c*: noun constants.
- *X*, *Y*, *Z*: variables for predicates.
- $P$, $Q$, $P_1$, … , $P_{15}$: unary predicate symbols.
- *A*, *B*, *C*, *D*: symbols for propositions.
- ~, ∨, ∧, →, ↔: propositional connectives of negation, disjunction, conjunction, implication, bi-implication.
- ⊢: derivation.
- =: identity.
- >: greater than (see also footnote 29).
- ⊥: symbol for a contradiction (see RCo1 for a detailed explanation).

We will also use the following symbols:

- *Co*(.) = "conceivable that." If "*A*" is a proposition then "*Co*(*A*)" will in turn be a proposition (which we can also think of as a nullary predicate); if "*A*" is a formula with number *n* of free variables we can think of it as a *n*-ary predicate, and "*Co*(*A*)" will also be an *n*-ary predicate, which we add to our signature.
- *ex*(.) = "exists in reality" (predicate saturable by noun variables or constants).
- *co*(.) = "conceivable" (predicate saturable by noun variables or constants).

– $\forall x$; $\exists x$: universal and particular quantifiers. We want to make clear that for these quantifiers the variable "$x$" can also be substituted by empty noun constants, meaning that they do not denote something which really exists, for instance, "Pegasus" or "Nothing." Because of this, the quantifier "$\exists x$" should be read as "for some $x$" or "for something," and not as "there exists an $x$ such that…" This way, the proposition "$\sim ex$(Pegasus)" will be true (indeed, Pegasus does not exist in reality) and there will be no problem with propositions such as "$\exists x \sim ex(x)$," that is, "for some $x$, $x$ does not exist in reality."[22] We assume that with this interpretation the inference rules for quantifiers of classical logic still hold. There will be a reminder on this topic in 3.0.6.

### 3.0.3. Syntax and Semantics

In our formal analysis of the *unum argumentum* we will use the classical syntax and the semantics of classical predicate logic and the propositional calculus.

### 3.0.4. Axioms

The following axiom states that something corresponds to the description of IQM.

**Pr IQM)** $(\exists x)(\forall y) \sim Co(y > x)$ (*Something is such that a greater is not conceivable*)

In other words "for some $x$, for every $y$, a $y$ greater than $x$ is not conceivable." Note that Pr IQM:

– is not a definition expressed through material equivalence ($P \leftrightarrow Q$) (see Def GAC and Def INE below), but rather a sentence that describes an undetermined $x$;[23]

[22] The topic is of course linked to that of existence in logic exposed by W.O. Quine (*On What There Is*, in: W.O. Quine, *From a Logical Point of View*, Cambridge, MA, 1953, pp. 1–19). The literature on it is abundant: for an overview one can see, for example, M. Reicher, *Nonexistent Objects*, in: *The Stanford Encyclopedia of Philosophy* (Winter 2019 Edition), ed. E.N. Zalta, URL: https://plato.stanford.edu/archives/win2019/entries/nonexistent-objects/. For a rigorous exposition on the use of quantifiers extended to the case of empty terms, see C. Lejewski, *Logic and Existence*, "British Journal for the Philosophy of Science" 1954, Vol. 5, pp. 104–119; D.P. Henry, *Medieval Logic and Metaphysics*, London 1972, pp. 25–32.

[23] Here we follow T.A. Robinson and his detailed analysis (*A New Formalization of Anselm's Ontological Argument*, in *Philosophy Faculty Publications* 2004, Vol. 6, URL: http://digitalcommons.csbsju.edu/philosophy_pubs/6, pp. 3–5).

– is not a logical truth like "$A \rightarrow A$" ("if $A$ then $A$" is a tautology whose truth table is always true); for this reason, the *Proslogion* cannot be reduced to a purely logical dimension but, in this sense, *only if* a concretely empirical subject recognizes and proves[24] that Pr IQM makes sense, then he must necessarily accept all that follows;
– says nothing about the conceivability or non-conceivability of IQM, nor does it state that it is greater than something.

In the proofs we will use, like Anselm does, this description as an accepted truth (like an axiom), because its "poor" logical content could be accepted by everyone (see 3.1.4).

In sections 3.1.2–4 we will consider some other propositions that state that similar descriptions are satisfied by something, simply in order to show that from them one cannot derive the real existence of the objects described: unlike Pr IQM, such propositions are not axioms within our theory; rather, they only serve the purpose of showing the "strength" and unique character of Pr IQM.

**Ax Hier$_1$)** $(\forall x)(\forall y)\ \{[\sim ex(x) \land ex(y)\ ] \rightarrow y > x\}$

It is the first Hierarchy Axiom, according to which if $x$ does not exist in reality while $y$ does, then $y$ is greater than $x$.

In 3.2 and 3.3, seventeen similar Hierarchy Axioms will be introduced. Each of them thematizes a predicate that must be interpreted not simply as a property, but as a perfection that makes a being be greater than another. To make the text of the *Proslogion* even better as a theoretical path, in which the initial notion of IQM is gradually enriched with perfections, all these hierarchies could be defined in an almost recursive way, also referring to the perfections demonstrated in the previous steps, according to this axiom schema:

**Ax Hier$_n$)** $(\forall x)(\forall y)\ \{[P_1(x) \land P_1(y) \land \ldots \land P_{n-1}(x) \land P_{n-1}(y) \land \sim P_n(x) \land P_n(y)]$
$\rightarrow y > x\}$

That is: if $x$ and $y$ both enjoy the properties $P_1$-$P_{n-1}$, but only $y$ enjoys $P_n$, then $y$ will be greater than $x$. In this way, to prove Theorem $i$, the first step will be to recall the theorem in which the Hierarchy Axiom $i$-1 was used (see, for ex-

[24] For the concept of "to prove" as distinct from that of "to demonstrate," but fundamental for understanding the *Proslogion* as well, see G. Barzaghi, *La prova dell'esistenza di Dio. Il retroscena metafisico della dimostrazione*, "Aquinas" 2019, Vol. 62, No. 1–2, pp. 11–20.

ample, 3.3, Theor 3.1, footnote 46, line 1.1). Note that the rule works also in case $x$ lacks more than one of those predicates with respect to $y$,[25] and holds only for some properties listed in a definite order (so it does not hold for every property).[26]

### 3.0.5. Definitions

In sections 3.2 and 3.3 these definitions will be used:

**Def GAC)** $(\forall x)\ \{GAC(x) \leftrightarrow (\forall y)\ [(x \neq y) \wedge co(y)] \rightarrow x > y\}$
*(x is greater than any different conceivable if and only if for every different conceivable object, x is greater)*

**Def INE)** $(\forall x)\ \{INE(x) \leftrightarrow \sim Co[\sim ex(x)]\}$
*(x is inconceivable as non-existent in reality if and only if it is not conceivable that it does not exist in reality)*

### 3.0.6. Derivation Rules

We will use the classical rules for the quantifiers, where $A$ is a proposition containing the variable $x$ or the constant $a$:

**EE)** $(\exists x)\ A[x] \vdash A[a]$[27]
(Elimination of the particular quantifier, where "$a$" is a constant not previously used in the proof)[28]

25 In fact, if $x$ lacks $1 + n$ predicates with respect to $y$ then it lacks at least one, and so the conclusion "$y > x$" follows anyway. More formally:

| | |
|---|---|
| 1) $P(x) \wedge P(y) \wedge \sim[Q(x)] \wedge Q(y) \wedge \sim[R(x)] \wedge R(y)$ | hp |
| 2) $P(x) \sim P(y) \wedge \sim[Q(x)] \wedge Q(y)$ | 1, CP: $(A \wedge B \wedge C) \rightarrow (A \wedge B)$ |
| 3) $y > x$ | 2, Ax $Hier_1$ |

26 In this respect, we could hypothesize this case:

| | |
|---|---|
| 1) $P_1(x) \wedge P_1(y)$ | (In our article this means: $ex(x) \wedge ex(y)$) |
| 2) $P_2(x) \wedge \sim P_2(y)$ | (In our article this means: $\sim INE(x) \wedge INE(y)$) |
| 3) $\sim P_3(x) \wedge P_3(y)$ | (Where $P_3$ is a fixed property not defined) |

From this data it follows that:

4) $x > y$ For Ax $Hier_2$ and 1–2

However, it *does not* follow that

5) $y > x$

In fact, from 1 + 3 which is: $P_1(x) \wedge P_1(y) \wedge \sim P_3(x) \wedge P_3(y)$, it is not possible to apply any axiom of hierarchy to derive 5). In other words, for every $n$, Ax $Hier_n$ must be applied to $n$ fixed properties in a fixed order.

27 E.g. "EE $x/a$" means that in a line the particular quantifier "$(\exists x)$" has been removed and the variable "$x$" has been everywhere replaced with the constant "$a$."

28 We can make this rule work by making our theory into a Henkin theory the usual way, adding appropriate constants and axioms (L. Henkin, *The Completeness of the First-Order Functional*

**EU)** $(\forall x)\ A[x] \vdash A[a]$
(Elimination of the universal quantifier)

**IE)** $A[a] \vdash (\exists x)\ A[x]$
(Introduction of the particular quantifier)

**UE)** $(\forall x)\ A(x) \leftrightarrow \sim(\exists x) \sim A(x)$
(Definition of the universal quantifier by the existential quantifier)

Two additional rules governing conceivability are introduced, in the form of axiom schemata:

**RCo1)** For every $A$ and $A^*$ we have the corresponding axiom:
$A \rightarrow Co(A^*)$

where $A$ is any proposition and $A^*$ is a proposition such that $\{A^* \land O\} \nvdash \bot$ (from $A^*$ together with the strict order axioms[29] for "<" no contradiction follows) and differs from $A$ at most insofar as:

- one constant has been consistently replaced by a different constant;
- it is the negation of $A$ except for the fact that one constant has been consistently replaced by a different constant.

The sentence "$O$" expresses the fact that "<" is a strict order. By "$\nvdash \bot$" we mean "the negation of a tautology does not follow," or equivalently (by the Soundness and Completeness theorems) that the sentence or theory preceding the expression is satisfiable (that is, has a model). This is the sense in which we will use the terms "contradictory" or "self-contradictory" in the following.

For instance, if $A$ is the proposition "$P(a)$" interpreted as "Socrates runs" then it can be derived:

1 – "$Co[P(a)]$" = "It is conceivable that Socrates runs."[30]

3 – "$Co[P(b)]$" = "It is conceivable that Plato runs."

4 – "$Co[\sim P(b)]$" = "It is conceivable that Plato does not run."

But it cannot be derived, for example, "It is conceivable that Socrates runs and Socrates does not run."

---

*Calculus*, "The Journal of Symbolic Logic" 1949, Vol. 14, No. 3, pp. 159–166).

29 The strict order axioms are irreflexivity, antisymmetry, and transitivity. These mean respectively that "$a < a$" does not hold for any "$a$"; if "$a < b$" holds, then "$b < a$" does not hold; and if "$a < b$" and "$b < c$" hold, then "$a < c$" holds.

30 In Appendix B we will show that, if in RCo1 is included the possibility to conceive the negation of accepted propositions (RCo1.2: $Co[\sim P(a)]$ = "It is conceivable that Socrates does not run"), the theory is inconsistent.

**RCo2)** For any propositions $A$, $B$ and $C$ such that $A, B \vdash C$ and $\{C \wedge O\} \nvdash \bot$, we have the axiom:
$[Co(A) \wedge Co(B)] \rightarrow Co(C)$[31]

This rule is a sort of *modus ponens* based on the conceivability of the clauses of the antecedent, and states that if:

- "$A$ is conceivable" and "$B$ is conceivable";
- and from $A$ and $B$ one can derive $C$;
- and "$C$" is such that $\{C \wedge O\} \nvdash \bot$;

then we can derive "$C$ is conceivable."

For instance, if:

- "It is conceivable that Plato is the husband of Xanthippe" ($Co(A)$);
- and "It is conceivable that Plato is the teacher of Socrates" ($Co(B)$);
- and from "Plato is the husband of Xanthippe" ($B$) and "Plato is the teacher of Socrates" ($A$) derives "The husband of Xanthippe is the teacher of Socrates" ($C$);
- given that "The husband of Xanthippe is the teacher of Socrates" is not self-contradictory,

then it is derivable that "It is conceivable that the husband of Xanthippe is the teacher of Socrates" ($Co(C)$).

Essentially, in a deduction in which there are conceivable sentences, one can mentally cancel "*Co*" and hence derive a non-contradictory consequence, and then *must* add that this consequence is only conceivable (adding "*Co*" to what is derived).[32]

One might think that there is a vicious circle, since it seems that the description of RCo1 and RCo2 depends on whether no contradiction follows from a certain formula within the system, while the system itself is being described, among other things, by these axiom schemata. Actually, for RCo1 and RCo2, the fact that a certain formula constitutes or not an instance of the axiom schema depends only on the consistency of a single proposition, $A^* \wedge O$ or $C \wedge O$ respectively, con-

[31] We can of course take a tautology for $B$ and apply this rule just with $A$ and $C$. In other words, if $A \vdash C$ and $C$ is not self-contradictory: $Co(A) \rightarrow Co(C)$.

[32] These rules obviously do not exhaust the theme of conceivability, which, moreover, is a very recent field of study, not reducible to the modal concept of possibility (see T.S. Gendler, J. Hawthorne, eds., *Conceivability and Possibility*, Oxford 2002). In fact, since cats in our world are of different colours, it is possible to have a *different* world where there are only black cats, while it can be conceived that *in our world* cats are only black (even though this is false).

sidered in itself, and not together with the other axioms of our theory. This means that it is enough to produce a model for $\{A^* \wedge O\}$ or $\{C \wedge O\}$ to show that they do not produce contradictions (see Appendix A). In other words, the description depends on whether a contradiction follows or not from the theory that has *as its only axiom* $A^* \wedge O$ or $C \wedge O$ respectively, by applying the usual inference rules of classical first-order logic. Therefore, the nature of these axioms schemata depends on the system described in this paper only in the sense that it depends on its *inference rules*, which are the usual ones, while the system depends on these axioms only in the sense that they contribute to define the *theory*, which is a set of axioms, by being part of it.

Before ending this section, one last remark is needed. We have seen that the symbol "*Co*" can be applied both to propositions and predicates (including in the latter term any formula with some free variables). Consider the case of a proposition including a predicate, "*P*," and a constant, "*c*," "$P(c)$." We can apply the symbol "*Co*" to it and obtain "$Co[P(c)]$." We can also define the predicate "$Co(P)(x)$" and then substitute "*x*" with "*c*" to obtain "$Co(P)(c)$." It follows from our setting that these two expressions are formally distinct, but since we do not think that they express something really different about "*c*" we will assume their equivalence in the following *identification axiom* for the case described above:

**Ax ID)** $Co[P(c)] \leftrightarrow Co(P)(c)$

In the following, we will use this principle without explicit mention and drop the distinction in parentheses between the two expressions since they are for all purposes identical.

### 3.0.7. Formal Proofs

In this section, by "formal proof" we mean the derivation of a proposition from axioms or from theorems previously proved. The formal proofs will be carried out with the usual method: numbering the lines on the left, explaining how a certain line is derived from the previous lines indicating the rules and the axioms used. The tautologies of PC (expressed in the right part of the formal proofs) will be used as rules of derivation thanks to the Deduction Theorem.[33] Sometimes we will, with an abuse of notation, refer with "A, B → A ∧ B" to the derivation rule

[33] The implicit use of the Deduction Theorem (TD: $(A, B \vdash C)$ *if and only if* $A \vdash B \rightarrow C$) allows us to enunciate a theorem as a proposition (from left to right) and to use PC tautologies as deductive rules (from right to left).

that allows us to deduce from two propositions their conjunction. For a proof of the fact that the steps where we use RCo1 and RCo2 correspond indeed to instances of such axioms (that is, that the proposition inside "*Co*" in the conclusion is not *in itself* contradictory) we refer the reader to Appendix A.

## 3.1. IQM Exists in Reality (Ch. 2)

### 3.1.1. *Unum Argumentum*: Logical Structure

Below is the reasoning from 2.1 presented in a more formal way, with comments on the passages to the side and in the footnotes.

**Theor 1.1)** $(\exists x)\ \{(\forall y) \sim Co(y > x) \wedge ex(x)\}$[34]
*(Something than which a greater is not conceivable exists in reality)*

[34] Here is the detailed formal proof, with some notes of explanation:

| | | |
|---|---|---|
| 1.1) | $(\exists x)(\forall y) \sim Co(y > x)$ | Pr IQM |
| 1) | $(\forall y) \sim Co(y > a)$ | 1.1, EE $x/a$ |
| 2) | $\sim ex(a)$ | hp |
| 3.1) | $Co[(\forall y) \sim Co(y > b)]$ | 1, RCo1.3 $a/b$ (*In 1, thanks to RCo1-option 3, the constant a is changed into b and "Co" must be added on the left*) |
| 3.2) | $Co[\sim\sim ex(b)]$ | 2, RCo1.4 (*In 2, thanks to RCo1-option 4, the constant a is changed into b, the negation is added, and "Co" must be added on the left*) |
| 3.3) | $Co[ex(b)]$ | 3.2, RCo2, PC: $\sim\sim A \leftrightarrow A$ (*In 3.2, thanks to RCo2, "Co" is temporarily canceled, the rule of the double negation is applied, and "Co" must be added on the left*) |
| 3) | $Co\{(\forall y) \sim Co(y > b) \wedge ex(b)\}$ | 3.1–3.3, RCo2, PC: $A, B \rightarrow A \wedge B$ (*In 3.1 and in 3.3, thanks to RCo2, "Co" is temporarily canceled, the resulting propositions are unified in a conjunction thanks to PC, and "Co" must be added on the left*) |
| 4.1) | $Co[ex(b)]$ | 3, RCo2, PC: $(A \wedge B) \rightarrow B$ (*In 3, thanks to RCo2, "Co" is temporary canceled, "ex(b)" is detached thanks to PC, and "Co" must be added on the left*) |
| 4.2) | $Co[\sim ex(a)]$ | 2, RCo1.1 |
| 4.3) | $Co[\sim ex(a) \wedge ex(b)]$ | 4.2 + 4.1, RCo2, PC: $A, B \rightarrow A \wedge B$ (*From 4.2 and 4.1, thanks to RCo2, "Co" is temporarily canceled and, thanks to PC, the propositions are unified in a conjunction, and 'Co' must be added on the left*) |

In the following we summarize the formal proof, highlighting the six main points of chapter 2 of the *Proslogion* presented in 2.1:

1) $(\forall y) \sim Co(y > a)$ — Pr IQM, EE $x/a$
(*Something than which a greater is not conceivable exists in the intellect*)[35]

2) $\sim ex(a)$ — hp[36]
(*It is hypothesized that something than which a greater is not conceivable does not exist in reality*)

3) $Co\{(\forall y) \sim Co(y > b) \wedge ex(b)\}$ — 1, RCo1.3; 2, RCo1.4; RCo2, PC: $\sim\sim A \leftrightarrow A$, RCo2, PC: $A, B \rightarrow A \wedge B$
(*It is conceived something than which a greater is not conceivable which exists in reality*)

4) $Co(b > a)$ — 3, RCo2, PC: $(A \wedge B) \rightarrow B$; 2, RCo1.1; RCo2, PC: $A, B \rightarrow A \wedge B$; Ax $Hier_1$ EU; RCo1.1; RCo2 PC: $[(A \rightarrow B) \wedge A] \rightarrow B$

---

| | | |
|---|---|---|
| 4.4) | $[\sim ex(a) \wedge ex(b)] \rightarrow (b > a)$ | Ax $Hier_1$ EU $x/a, y/b$ |
| 4.5) | $Co\{[\sim ex(a) \wedge ex(b)] \rightarrow (b > a)\}$ | 4.4., RCo1.1 |
| 4) | $Co(b > a)$ | 4.3, 4.5., RCo2 PC: $[(A \rightarrow B) \wedge A] \rightarrow B$ (*In 4.3–4.5, thanks to RCo2, "Co" is temporarily canceled and, thanks to PC, "b > a" can be derived and "Co" must be added*) |
| 5) | $\sim Co(b > a)$ | 1, EU $y/b$ |
| 6.1) | $\sim\sim ex(a)$ | 2, 4 + 5, PC: $[A \rightarrow (B \wedge \sim B)] \rightarrow \sim A$ (*From hypothesis 2, the contradictory propositions 4 + 5 follow, so thanks to PC, 2 must be negated*) |
| 6.2) | $ex(a)$ | 6.1, PC: $\sim\sim A \leftrightarrow A$ |
| 6.3) | $(\forall y) \sim Co(y > a) \wedge ex(a)$ | 1, 6.2, PC: $A, B \rightarrow A \wedge B$ |
| 6) | $(\exists x) \{(\forall y) \sim Co(y > x) \wedge ex(x)\}$ | 6.3, IE $a/x$ |

Note that 4.1 is identical to 3.3; however, we prefer to derive it from 3) since this is more faithful to Anslem's text.

35 Some authors define existence in the intellect as a specific predicate that applies to individual contents of a proposition (see R.S. Silvestre, *On the Logical Formalization of Anselm's Ontological Argument*, "Revista Brasileira de Filosofia da Religião" 2015, Vol. 2, No. 2, p. 148 ff.; J.-P. Desclés, *A Logical Analysis of the Anselm's Unum Argumentum (from Proslogion)*, "Logica Universalia" 2017, Vol. 11, p. 107; T.A. Robinson, *A New Formalization of Anselm's Ontological Argument*, op. cit., p. 2). On the other hand, some have made existence in the intellect coincide with the particular quantifier (for a discussion, see G. Eder, E. Ramharter, *Formal Reconstructions of St. Anselm's Ontological Argument*, "Synthese" 2015, Vol. 192, No. 9, pp. 2800–2802).

36 See 3.0.2 and footnote 22 for the meaning of "$\sim ex(a)$," from which, thanks to IE, "$(\exists x) \sim ex(x)$" immediately follows.

(*This something than which a greater is not conceivable that exists in reality, is conceivably greater than that something such that a greater is not conceivable and that does not exist in reality*)[37]

5) $\sim Co(b > a)$ — 1, EU $y/b$

(*If a is something than which a greater is not conceivable, then it also cannot be conceived that b is greater than a*)

6) $(\exists x)\ \{(\forall y) \sim Co(y > x) \wedge ex(x)\}$ — 2, 4 + 5, PC: $[A \rightarrow (B \wedge \sim B)] \rightarrow \sim A$; PC: $\sim\sim A \leftrightarrow A$; PC: $A, B \rightarrow A \wedge B$; IE $a/x$

(*4 and 5 are contradictory, therefore hypothesis 2, from which this contradiction derives, must be denied, and therefore IQM exists in reality*)

### 3.1.2. *Unum Argumentum* and Specific Entities

Regarding the difference between IQM and the example of the fabulous island, we can thus describe the concept of "the island than which a greater island is not

[37] This conclusion derives only from conceiving "$ex(b)$" and not from other conditions under which $b$ is conceived. This can be seen as a limitation of this formalization: in fact, to prove that IQM exists in reality, just assume that it does not exist and conceive that there is at least one existing entity: but we cannot derive Theor 2.1 and following. However, in Theor 1.1, line 3), it is specifically recalled that this $b$ must also satisfy the IQM condition because this better expresses the text of Anselm, which is the aim of this article.

In order to make the proof depend on the conceivability of an existent IQM (and not just on that of a generic existent object), one could define the predicate "IQM( )" ("Being IQM") and change the hierarchy axiom as follows:

| | |
|---|---|
| Def IQM) | $(\forall x)\ \{\mathrm{IQM}(x) \leftrightarrow (\forall y) \sim Co(y > x)\}$ |
| Ax Hier$_{1.2}$) | $(\forall x)\ (\forall y)\ \{[\mathrm{IQM}(x) \wedge \mathrm{IQM}(y) \wedge \sim ex(x) \wedge ex(y)] \rightarrow y > x\}$ |

So that, in proof 1.1 and by analogy in the ones after that, only the following lines should be changed:

| | | |
|---|---|---|
| 4.1) | $(\forall y) \sim Co(y > a) \wedge \sim ex(a)$ | 1, 2 PC: $A, B \rightarrow A \wedge B$ |
| 4.2) | $Co[(\forall y) \sim Co(y > a) \wedge \sim ex(a)]$ | 4.1, RCo1.1 |
| 4.3) | $Co[(\forall y) \sim Co(y > a) \wedge (\forall y) \sim Co(y > b) \wedge \sim ex(a) \wedge ex(b)]$ | 4.2 + 3, RCo2, PC: $A, B, C, D \rightarrow A \wedge C \wedge B \wedge D$ |
| 4.4) | $Co[\mathrm{IQM}(a) \wedge \mathrm{IQM}(b) \wedge \sim ex(a) \wedge ex(b)]$ | 4.3, RCo2; Def IQM, ELU $x/a$, Def IQM, ELU $x/b$, PC: $[(A \wedge C \wedge B \wedge D) \wedge (A \leftrightarrow E) \wedge (B \leftrightarrow F)] \rightarrow (E \wedge F \wedge B \wedge D)$ |
| 4.5) | $[\mathrm{IQM}(a) \wedge \mathrm{IQM}(b) \wedge \sim ex(a) \wedge ex(b)] \rightarrow (b > a)$ | Ax Hier$_{1.2}$ EU $x/a, y/b$ |

Note also that in Theor 1.1 from 4) we can directly derive the contradictory of 1):

| | | |
|---|---|---|
| 4.1) | $(\exists y)\ Co(y > a)$ | 4, IE |
| 4.2) | $\sim(\forall y) \sim Co(y > a)$ | 4.1), UE |

conceivable" (where "$P(.)$" is a predicate whose interpretation here is "to be an island"):

**Pr FIQM)** $(\exists x)(\forall y) \sim Co[P(x) \wedge P(y) \wedge y > x]$

Now, *if* this sentence is assumed to be true we can apply the demonstration strategy used in Theor 1.1. (here and some other times later in this paper we will use again the constant symbols "$a$" and "$b$" for simplicity's sake, even though they are not meant to denote the same objects in different proofs), but no contradictions follow, as this sketch of formal proof shows:[38]

| | |
|---|---|
| 1) $(\forall y) \sim Co[P(a) \wedge P(y) \wedge y > a]$ | hp Pr FIQM, EE $x/a$ |
| 2) $\sim ex(a)$ | hp |
| 3) $Co\{(\forall y) \sim \mathrm{Co}[P(b) \wedge P(y) \wedge y > b] \wedge ex(b)\}$ | 1 + RCo1.3, 2 + RCo1.4, RCo2, PC: $\sim\sim A \leftrightarrow A$; $A, B \rightarrow (A \wedge B)$ |
| 4) $Co(b > a)$ | 3, RCo2, PC: $(A \wedge B) \rightarrow B$; 2, RCo1.1; RCo2, PC: $\sim\sim A \leftrightarrow A$, Ax $\mathrm{Hier}_1$ EU $x/a$, $y/b$; RCo1.1; RCo2, PC: $[(A \rightarrow B) \wedge A] \rightarrow B$ |
| 5) $\sim Co[P(a) \wedge P(b) \wedge b > a]$ | 1, EU $y/b$ |

Unlike Theor 1.1) the formal proof stops here because from 4), nothing follows, not being a negation contradicting 5), as clearly seen in the last two lines. The demonstration only tells us that, *if* the island than which a greater island is

[38] Here is the detailed formal proof:

| | | |
|---|---|---|
| 1.1) | $(\exists x)(\forall y) \sim Co[P(x) \wedge P(y) \wedge y > x]$ | hp Pr FIQM |
| 1) | $(\forall y) \sim Co[P(a) \wedge P(y) \wedge y > a]$ | 1.1, EE $x/a$ |
| 2) | $\sim ex(a)$ | hp |
| 3.1) | $Co[(\forall y) \sim Co[P(b) \wedge P(y) \wedge y > b]]$ | 1, RCo1.3 $a/b$ |
| 3.2) | $Co[\sim\sim ex(b)]$ | 2, RCo1.4 |
| 3.3) | $Co[ex(b)]$ | 3.2, RCo2, PC: $\sim\sim A \leftrightarrow A$ |
| 3) | $Co\{(\forall y) \sim Co[P(b) \wedge \mathrm{P}(y) \wedge y > b] \wedge ex(b)\}$ | 3.1–3.3, RCo2, PC: $A, B \rightarrow A \wedge B$ |
| 4.1) | $Co[ex(b)]$ | 3, RCo2, PC: $(\mathrm{A} \wedge \mathrm{B}) \rightarrow \mathrm{B}$ |
| 4.2) | $Co[\sim ex(a)]$ | 2, RCo1.1 |
| 4.3) | $Co[\sim ex(a) \wedge ex(b)]$ | 4.2 + 4.1, RCo2, PC: $\mathrm{A}, \mathrm{B} \rightarrow \mathrm{A} \wedge \mathrm{B}$ |
| 4.4) | $[\sim ex(a) \wedge ex(b)] \rightarrow (b > a)$ | Ax $\mathrm{Hier}_1$ EU $x/a$, $y/b$ |
| 4.5) | $Co\{[\sim ex(a) \wedge ex(b)] \rightarrow (b > a)\}$ | 4.4., RCo1.1 |
| 4) | $Co(b > a)$ | 4.3, 4.5., RCo2 PC: $[(A \rightarrow B) \wedge A] \rightarrow B$ |
| 5) | $\sim Co[P(a) \wedge P(b) \wedge b > a]$ | 1, EU $y/b$ |

not conceivable does not exist, then it is possible to conceive a greater entity than it (4), which does not cause any problems. In other words, from the non-existence of this fabulous island, it only follows that a greater being can be conceived, and not also that both are islands.

### 3.1.3. *Unum Argumentum* and the Greatest

The proof of the *Proslogion* chapter 2 does not even apply to those notions that hinge on the notion of greatest, which could be expressed by the following propositions, in analogy with Pr IQM:

**Pr GE)** $(\exists x)(\forall y)\ [(y \neq x) \rightarrow (x > y)]$
*(Something greater than everything else)*[39]

**Pr NG)** $(\exists x)(\forall y) \sim(y > x)$
*(Something not having a greater)*

**Pr CG)** $(\exists x)(\forall y)\ Co(x > y)$
*(Something conceivable as greater than everything)*

**Pr CNG)** $(\exists x)(\forall y)\ Co[\sim(y > x)]$
*(Something conceivable as not having a greater)*

On the one hand, in fact, the first two propositions are much stronger than Pr IQM because they affirm a statement and do not indicate it as merely conceivable, so they would not be as plausible axioms as Pr IQM. And, moreover, *if*, for example, Pr GE) is assumed to be true of something, then we cannot derive that the greater than everything else exists in reality, as this sketch of formal proof shows:[40]

[39] Here, if we were to use the self-contradictory premise $(\exists x)(\forall y)\ (x > y)$, the existence in reality of something which satisfies the description would of course follow since from a contradiction everything follows. Pr CG) does not lead to the same problem since we are requiring that for every y it is merely conceivable that x is greater than y. We could also infer that $Co(a > a)$, but notice that even this is not a contradiction since our theory does not exclude conceiving a contradictory statement, but only that conceiving such a statement be the end-point of an application of RCo1 or RCo2. There does not seem to be any philosophical problem in conceiving a contradictory proposition, as the case of conceiving a mathematical conjecture either as true or false exemplifies. From the quotation in 2.0.c it seems that Anselm might have wanted to exclude from conceivability only contradictions in the strict sense, i.e., conjunctions of a proposition and its negation.

[40] Here is the detailed formal proof:

| | | |
|---|---|---|
| 1.1) | $(\exists x)(\forall y)\ [(y \neq x) \rightarrow (x > y)]$ | hp Pr GE |
| 1) | $(\forall y)\ (y \neq a) \rightarrow (a > y)$ | 1.1, EE *x/a* |

| | |
|---|---|
| 1) $(\forall y)\ (y \neq a) \rightarrow (a > y)$ | hp Pr GE, EE $x/a$ |
| 2) $\sim ex(a)$ | hp |
| 3) $Co\{(\forall y)\ [(y \neq b) \rightarrow (b > y)] \land ex(b)\}$ | 1, RCo1.3 $a/b$; 2, RCo1.4; RCo2, PC: $\sim\sim A \leftrightarrow A$; RCo2, PC: $A, B \rightarrow A \land B$ |
| 4) $Co(b > a)$ | 3, RCo2, PC: $(A \land B) \rightarrow$ B; 2, RCo1.1; RCo2, PC: $A, B \rightarrow A \land B$; Ax $Hier_1$ EU $x/a$, $y/b$; RCo1.1; RCo2, PC: $[(A \rightarrow B) \land A] \rightarrow B$ |
| 5) $(b \neq a) \rightarrow (a > b)$ | 1, EU $y/b$ |

The formal proof ends here, since 4) and 5) do not contradict each other: if "*a*" is greater than everything else and does not exist in reality, then it follows "only" that is conceivable something "*b*" greater than "*a*," but if "*a*" is different from "*b*" then "*a*" is still greater than "*b*" in reality.

Pr CG), on the other hand, given that it refers to the sphere of conceivability, could be accepted as an axiom. However, unlike IQM, this description, if accepted as true of something, does not negate "*Co*," so its particularization ("$Co(a > b)$": line 5 in Theor 1.1) will not lead to a contradiction by denying sentences obtained using RCo1–2 (which always begin with "*Co*")[41]:

| | | |
|---|---|---|
| 2) | $\sim ex(a)$ | hp |
| 3.1) | $Co\{(\forall y)\ [(y \neq b) \rightarrow (b > y)]\}$ | 1, RCo1.3 $a/b$ |
| 3.2) | $Co[\sim\sim ex(b)]$ | 2, RCo1.4 |
| 3.3) | $Co[ex(b)]$ | 3.2, RCo2, PC: $\sim\sim A \leftrightarrow A$ |
| 3) | $Co\{(\forall y)\ [(y \neq b) \rightarrow (b > y)\ ] \land ex(b)\}$ | 3.1–3.3, RCo2, PC: $A, B \rightarrow A \land B$ |
| 4.1) | $Co[ex(b)]$ | 3, RCo2, PC: $(A \land B) \rightarrow B$ |
| 4.2) | $Co[\sim ex(a)]$ | 2, RCo1.1 |
| 4.3) | $Co[\sim ex(a) \land ex(b)]$ | 4.2 + 4.1, RCo2, PC: $A, B \rightarrow A \land B$ |
| 4.4) | $[\sim ex(a) \land ex(b)] \rightarrow (b > a)$ | Ax $Hier_1$ EU $x/a$, $y/b$ |
| 4.5) | $Co\{[\sim ex(a) \land ex(b)\ ] \rightarrow (b > a)\}$ | 4.4., RCo1.1 |
| 4) | $Co(b > a)$ | 4.3, 4.5., RCo2 PC: $[(A \rightarrow B) \land A] \rightarrow B$ |
| 5) | $(b \neq a) \rightarrow (a > b)$ | 1, EU $y/b$ |

41 Here is the detailed formal proof:

| | | |
|---|---|---|
| 1.1) | $(\exists x)(\forall y)\ Co(x > y)$ | hp Pr CG |
| 1) | $(\forall y)\ Co(a > y)$ | 1.1, EE $x/a$ |
| 2) | $\sim ex(a)$ | hp |
| 3.1) | $Co[(\forall y)\ Co(b > y)]$ | 1, RCo1.3 $a/b$ |
| 3.2) | $Co[\sim\sim ex(b)]$ | 2, RCo1.4 |
| 3.3) | $Co[ex(b)]$ | 3.2, RCo2, PC: $\sim\sim A \leftrightarrow A$ |
| 3) | $Co\{(\forall y)\ Co(b > y) \land ex(b)\}$ | 3.1–3.3, RCo2, PC: $A, B \rightarrow A \land B$ |

| | |
|---|---|
| 1) $(\forall y)\ Co(a > y)$ | hp Pr CG, EE $x/a$ |
| 2) $\sim ex(a)$ | hp |
| 3) $Co[(\forall y)\ Co(b > y) \wedge ex(b)]$ | 1+ RCo1.3, 2 + RCo1.4, RCo2, PC: $\sim\sim A \leftrightarrow A$; $A, B \rightarrow (A \wedge B)$ |
| 4) $Co(b > a)$ | 3, RCo2, PC: $(A \wedge B) \rightarrow B$; 2, RCo1.1; RCo2, PC: $\sim\sim A \leftrightarrow A$, Ax Hier1 EU $x/a$, $y/b$; RCo1.1; RCo2, PC: $[(A \rightarrow B) \wedge A] \rightarrow B$ |
| 5) $Co(a > b)$ | 1, EU $y/b$ |

The formal proof ends here, since 4) and 5) do not contradict each other: if "*a*" is conceivable as greater than everything and does not exist in reality, then "*b*" is conceivable as greater than "*a*," but "*a*" is also conceivable as greater than "*b*" (see RCo1–2; 2.1).

It is also worth noting that from Pr CNG) no contradiction follows since, with the same strategy, line 4 ("$Co(b > a)$") would not contradict the particularization of Pr CNG) ("$Co[\sim(b > a)]$"), being two contradictory propositions separately conceivable, even though not conceivable together (see RCo1–2; 2.1).

#### 3.1.4. *Unum Argumentum* and the Most Perfect Entity (*Ens Perfectissimum*)

Anselm's proof does not work even if we base it on the idea of the most perfect entity (or *ens perfectissimum*), which we could define as follows:

**Def *EP*)** $(\forall x)\ \{EP(x) \leftrightarrow (\forall X) \sim[PP(X) \wedge \sim X(x)]\}$
*(The ens perfectissimum is the one who does not lack any perfection)*

That is: $x$ is a most perfect entity ("$EP(x)$") if, and only if, there cannot be a predicate that is a positive property ("$PP(X)$") that is not enjoyed by $x$ ("$\sim X(x)$"). Now, since "*ex*" (existence in reality) is certainly a positive property, we should accept this axiom:

| | | |
|---|---|---|
| 4.1) | $Co[ex(b)]$ | 3, RCo2, PC: $(A \wedge B) \rightarrow B$ |
| 4.2) | $Co[\sim ex(a)]$ | 2, RCo1.1 |
| 4.3) | $Co[\sim ex(a) \wedge ex(b)]$ | 4.2 + 4.1, RCo2, PC: $A, B \rightarrow A \wedge B$ |
| 4.4) | $[\sim ex(a) \wedge ex(b)] \rightarrow (b > a)$ | Ax $Hier_1$ EU $x/a$, $y/b$ |
| 4.5) | $Co\{[\sim ex(a) \wedge ex(b)] \rightarrow (b > a)\}$ | 4.4., RCo1.1 |
| 4) | $Co(b > a)$ | 4.3, 4.5., RCo2 PC: $[(A \rightarrow B) \wedge A] \rightarrow B$ |
| 5) | $Co(a > b)$ | 1, EU $y/b$ |

**Ax *ex*)** $PP(ex)$

From this it follows that, if something is a most perfect entity, then it must exist:

| | |
|---|---|
| 1) $\sim(\forall x)\ [EP(x) \rightarrow ex(x)]$ | hp |
| 2) $(\exists x) \sim[EP(x) \rightarrow ex(x)]$ | 1, UE |
| 3) $(\exists x)\ [EP(x) \wedge \sim ex(x)]$ | 2, PC: $\sim(A \rightarrow B) \leftrightarrow (A \wedge \sim B)$ |
| 4) $EP(a) \wedge \sim ex(a)$ | 3, EE $x/a$ |
| 5) $EP(a)$ | 4, PC: $(A \wedge B) \rightarrow A$ |
| 6) $\sim ex(a)$ | 4, PC: $(A \wedge B) \rightarrow B$ |
| 7) $EP(a) \leftrightarrow (\forall X) \sim[PP(X) \wedge \sim X(a)]$ | Def $EP$, EU $x/a$[42] |
| 8) $EP(a) \rightarrow \sim[PP(ex) \wedge \sim ex(a)]$ | 7, EU $X/ex$ |
| 9) $\sim[PP(ex) \wedge \sim ex(a)]$ | 8, 5, PC: $[(A \rightarrow B) \wedge A] \rightarrow B$ |
| 10) $\sim PP(ex) \vee ex(a)$ | 9, PC: $\sim(A \wedge \sim B) \leftrightarrow (\sim A \vee B)$ |
| 11) $ex(a)$ | 10, Ax *ex*), PC: $[(\sim A \vee B) \wedge A] \rightarrow B$ |
| 12) $(\forall x)\ [EP(x) \rightarrow ex(x)]$ | 1, 11 + 6, PC: $[\sim A \rightarrow (B \wedge \sim B)] \rightarrow A$ |

But this certainly does not prove that an *EP* exists in reality!

Moreover, the actual perfection of *EP* depends (if the quantifiers on properties are interpreted as ranging only over really existent things) on the positive properties present in one's ontology. For example, a nominalist might think that only instantiated properties exist, so that in a world with only two individuals "*a*" and "*b*" such that "*a*" is beautiful but not intelligent and "*b*" is intelligent but not beautiful, the perfect entity would be just an entity that is both beautiful and intelligent.

However, the existence of an *EP* is not demonstrated even if the concept is extended to the conceivability of a most perfect entity which would, therefore, be more easily acceptable as existing, at least in the intellect (similarly to Pr IQM):

**Pr CEP)** $(\exists x)\ \{Co[(\forall X) \sim(PP(X) \wedge \sim X(x)]\}$
*(Something conceivable as not lacking any perfection)*

[42] In this section we have used the rules IE and UE also for variables for predicates.

In fact, let's assume this description as true of something and try to follow a similar path:

| | |
|---|---|
| 1) $(\exists x)\ \{Co[(\forall X)\ \sim PP(X) \wedge \sim X(x)]\}$ | hp Pr CEP |
| 2) $Co[(\forall X)\ \sim PP(X) \wedge \sim X(a)]$ | 1, EE $x/a$) |
| 3) $\sim ex(a)$ | hp |
| 4) $PP(ex)$ | Ax $ex$) |
| 5) $PP(ex) \wedge \sim ex(a)$ | 4, 3, PC: $A, B \rightarrow A \wedge B$ |
| 6) $(\exists X)\ PP(X) \wedge \sim X(a)$ | 5, IE $ex/X$ |
| 7) $\sim(\forall X)\ \sim[PP(X) \wedge \sim X(a)]$ | 6, UE |

Therefore, *if* it is conceivable that something does not lack any perfection and this thing does not exist then it *is not* something that does not lack any perfection: but this is not a contradiction.

It must be noted that in this section we have used the rules IE and UE also for predicates; however, outside of this section, which is not part of our attempt to formalize the *Proslogion*, we do not employ second-order logic. The theory we develop in sections 3.0.4–5 is entirely first-order.

### 3.2. IQM Is the Best of All, Such That Nothing Better Is Conceivable (*Summum*, *Melius*: Ch. 3–14)

To express more formally the second speculative moment in the *Proslogion*, in which IQM is understood as the better, we will use the same line of argument of Theor 1.1, but new hierarchal axioms are gradually introduced that will then be used to go from line 3 to 4.

Coming to the second positive property that is demonstrated about IQM after its existence, it must be admitted that if something exists and is inconceivable as non-existent in reality ("INE($x$)": see 3.0.5), it will be greater than what exists and is conceivable as non-existent in reality,[43] meaning:

[43] Henry instead explains the passage through modal notions such as "It is not possible not to conceive that" (D.P. Henry, *The Logic of St. Anselm*, Oxford 1993, pp. 145–148). He also examines Anselm's ontological argument in *Medieval Logic and Metaphysics*, op. cit., pp. 101–117: in this work Henry uses Leśniewski's logical system (see J. Slupeki, *Leśniewski Calculus of Name*,

**Ax Hier$_2$)** $(\forall x)(\forall y)\ \{[ex(x) \land ex(y) \land \sim\mathrm{INE}(x) \land \mathrm{INE}(y)] \rightarrow y > x\}$

Then using Theor 1.1 and the same strategy, it follows that "*a*" *is* inconceivable as non-existent in reality.

**Theor 2.1)** $(\exists x)\ \{(\forall y) \sim Co(y > x) \land ex(x) \land \mathrm{INE}(x)\}$[44]
*(Something than which a greater is not conceivable exists in reality and is inconceivable as non-existent in reality)*

| | |
|---|---|
| 1) $(\forall y) \sim \mathrm{Co}(y > a) \land ex(a)$ | Theor 1.1, EE $x/a$ |
| 2) $\sim\mathrm{INE}(a)$ | hp[45] |
| 3) $Co\{(\forall y) \sim Co(y > b) \land ex(b) \land \mathrm{INE}(b)\}$ | 1 + RCo 1.3, 2 + RCo 1.4, RCo2, PC: $\sim\sim A \leftrightarrow A$; A, B $\rightarrow$ A $\land$ B |

in: *Leśniewski's System: Ontology and Mereology*, eds. J. Srzeduick, V. Rickey, Boston–Lancaster 1984, pp. 59–122).

44 Here is the detailed formal proof:

| | | |
|---|---|---|
| 1.1) | $(\exists x)\ \{(\forall y) \sim Co(y > \mathrm{x}) \land ex(\mathrm{x})\}$ | Theor 1.1. |
| 1) | $(\forall y) \sim Co(y > a) \land ex(a)$ | 1.1, EE $x/a$ |
| 2) | $\sim\mathrm{INE}(a)$ | hp |
| 3.1) | $Co[(\forall y) \sim Co(y > b) \land ex(b)]$ | 1, RCo1.3 $a/b$ |
| 3.2) | $Co[\sim\sim\mathrm{INE}(b)]$ | 2, RCo1.4 |
| 3.3) | $Co[\mathrm{INE}(b)]$ | 3.2, RCo2, PC: $\sim\sim A \leftrightarrow A$ |
| 3) | $Co\{[(\forall y) \sim Co(y > b) \land ex(b)] \land \mathrm{INE}(b)\}$ | 3.1–3.3, RCo2, PC: $A, B \rightarrow A \land B$ |
| 4.1) | $Co[ex(b) \land \mathrm{INE}(b)]$ | 3, RCo2, PC: $[(A \land B) \land C] \rightarrow B \land C$ |
| 4.2) | $Co[\sim\mathrm{INE}(a)]$ | 2, RCo1.1 |
| 4.3.1) | $Co[(\forall y) \sim Co(y > a) \land ex(a)]$ | 1, RCo1.1 |
| 4.3) | $Co[ex(a) \land \sim\mathrm{INE}(a) \land ex(b) \land \mathrm{INE}(b)]$ | 4.3.1 + 4.2 + 4.1, RCo2, PC: $[(A \land B), C, D] \rightarrow (B \land C \land D)$ |
| 4.4) | $[ex(a) \land ex(b) \land \sim\mathrm{INE}(a) \land \mathrm{INE}(b)] \rightarrow (b > a)$ | Ax Hier$_2$ EU x/$a$, $y/b$ |
| 4.5) | $Co\{[ex(a) \land ex(b) \land \sim\mathrm{INE}(a) \land \mathrm{INE}(b)] \rightarrow (b > a)\}$ | 4.4., RCo1.1 |
| 4) | $Co(b > a)$ | 4.3, 4.5., RCo2 PC: $\{[(A \land B) \rightarrow C] \land (B \land A)\} \rightarrow C$ |
| 5.0) | $\sim Co(b > a) \land ex(a)$ | 1, EU $y/b$; |
| 5) | $\sim Co(b > a)$ | 5.0; RCo2, PC: $(A \land B \land C) \rightarrow A$ |
| 6.1) | $\sim\sim\mathrm{INE}(a)$ | 2, 4 + 5, PC: $[A \rightarrow (B \land \sim B)] \rightarrow \sim A$ |
| 6.2) | $\mathrm{INE}(a)$ | 6.1, PC: $\sim\sim A \leftrightarrow A$ |
| 6.3) | $(\forall y) \sim Co(y > a) \land ex(a) \land \mathrm{INE}(a)$ | 1, 6.2, PC: $A, B \rightarrow A \land B$ |
| 6) | $(\exists x)\{(\forall y) \sim Co(y > x) \land ex(x) \land \mathrm{INE}(x)\}$ | 6.3, IE $a/x$ |

45 In connection with the theme of conceivability, it is important to emphasize that from "$\sim\mathrm{INE}(a)$" can be derived "$Co[\sim ex(a)]$," which does not contradict "$ex(a)$" (Theor 1.1; see 2.0.b).

| | |
|---|---|
| 4) $Co(b > a)$ | 3, RCo2, PC: $[(A \wedge B) \wedge C] \rightarrow B \wedge C$; 2, RCo1.1; 1, RCo1.1; RCo2, PC: $\sim\sim A \leftrightarrow A$, Ax Hier2 EU $x/a$, $y/b$; RCo1.1; RCo2, PC: $[(A \rightarrow B) \wedge A] \rightarrow B$ |
| 5) $\sim Co(b > a)$ | 1, EU $y/b$, PC: $(A \wedge B) \rightarrow A$ |
| 6) $(\exists x)\ \{(\forall y) \sim Co(y > x) \wedge ex(x) \wedge \mathrm{INE}(x)\}$ | 2, 4 + 5, PC: $[A \rightarrow (B \wedge \sim B)] \rightarrow \sim A$; PC: $\sim\sim A \leftrightarrow A$; + 1, PC: $A, B \rightarrow A \wedge B$; IE $a/x$ |

Hence the series of theorems continues in the same way until all the others fourteen features listed in 2.2 are demonstrated, so IQM will be understood to be the *Melius*.

### 3.3. IQM Is Not Conceivable (*Maius Quam Cogitari Possit*: Ch. 15)

This is the third speculative moment of the *Proslogion*, in which, with the same strategy, it is shown that something than which a greater is not conceivable, besides existing in reality and being the Best, is also Greater than every conceivable. First of all, we have to use Def GAC) (*The greater of any conceivable:* see section 3.0.5). Then it is established, through a new hierarchy axiom, that if something shares the fifteen properties previously demonstrated for IQM and is also greater than any different conceivable, then it is greater than that which has the fifteen properties but is not greater than any different conceivable:

**Ax Hier$_{16}$)** $(\forall x)(\forall y)\ \{[ex(x) \wedge ex(y) \wedge \mathrm{INE}(x) \wedge \mathrm{INE}(y) \wedge \ldots \wedge \sim\mathrm{GAC}(x) \wedge \mathrm{GAC}(y)] \rightarrow y > x\}$

Finally, using the usual deductive strategy, it is possible to demonstrate that *something than which a greater is not conceivable*, exists in reality, is inconceivable as non-existent in reality, …., and *is greater than any different conceivable*:

**Theor 3.1)** $(\exists x)\ [(\forall y) \sim Co(y > x) \wedge ex(x) \wedge \mathrm{INE}(x) \wedge \ldots \wedge P_{15}(x) \wedge \mathrm{GAC}(x)]$[46]

[46] Here is the detailed formal proof:

| | | |
|---|---|---|
| 1.1) | $(\exists x)\ \{(\forall y) \sim Co(y > x) \wedge ex(x) \wedge \mathrm{INE}(x) \wedge \ldots \wedge P_{15}(x)\}$ | Theor 2.1 and ff. |
| 1) | $(\forall y) \sim Co(y > a) \wedge ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a)$ | 1.1, EE $x/a$ |

Not only that: as mentioned in 2.1.2 and 3.1.2, chapter 15 is in fact liable to an even more profound and "upsetting" reading, on the basis of which the *id quo maius cogitari non possit* must also be non-conceivable: this result is only possible if, as has been demonstrated, a definition of IQM is used which does not imply its conceivability.[47] To demonstrate this theorem, which is set out in an extremely concise way by Anselm, it is possible to use the usual strategy, based on a new hierarchy axiom:

**Ax Hier$_{17}$)** $(\forall x)(\forall y)\ \{[ex(x) \wedge ex(y) \wedge \ldots \wedge \mathrm{GAC}(x) \wedge \mathrm{GAC}(y) \wedge co(x) \wedge \sim co(y)] \rightarrow y > x\}$

| | | |
|---|---|---|
| 2) | $\sim \mathrm{GAC}(a)$ | hp |
| 3.1) | $Co[(\forall y) \sim Co(y > b) \wedge ex(b) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(b)]$ | 1, RCo1.3 *a/b* |
| 3.2) | $Co[\sim\sim \mathrm{GAC}(b)]$ | 2, RCo1.4 |
| 3.3) | $Co[\mathrm{GAC}(b)]$ | 3.2, RCo2, PC: $\sim\sim A \leftrightarrow A$ |
| 3) | $Co\{[(\forall y) \sim Co(y > b) \wedge ex(b) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(b)] \wedge \mathrm{GAC}(b)\}$ | 3.1–3.3, RCo2, PC: $A, B \rightarrow A \wedge B$ |
| 4.1) | $Co\ [ex(b) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(b) \wedge \mathrm{GAC}(b)]$ | 3, RCo2, PC: $[(A \wedge B) \wedge C] \rightarrow (B \wedge C)$ |
| 4.2) | $Co[\sim \mathrm{GAC}(a)]$ | 2, RCo1.1 |
| 4.3.1) | $Co[(\forall y) \sim Co(y > a) \wedge ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a)]$ | 1, RCo1.1 |
| 4.3) | $Co[ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a) \wedge \sim \mathrm{GAC}(a) \wedge ex(b) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(b) \wedge \mathrm{GAC}(b)]$ | 4.3.1 + 4.2 + 4.1, RCo2, PC: $[(A \wedge B), C, D] \rightarrow (B \wedge C \wedge D)$ |
| 4.4) | $[ex(a) \wedge ex(b) \wedge \mathrm{INE}(a) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(a) \wedge P_{15}(b) \wedge \ldots \wedge \sim \mathrm{GAC}(a) \wedge \mathrm{GAC}(b)] \rightarrow (b > a)$ | Ax Hier$_{26}$ EU *x/a*, *y/b* |
| 4.5) | $Co\{[ex(a) \wedge ex(b) \wedge \mathrm{INE}(a) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(a) \wedge P_{15}(b) \wedge \sim \mathrm{GAC}(a) \wedge \mathrm{GAC}(b)] \rightarrow (b > a)\}$ | 4.4., RCo1.1 |
| 4) | $Co(b > a)$ | 4.3, 4.5., RCo2 PC: $\{[(A \wedge B) \rightarrow C] \wedge (B \wedge A)\} \rightarrow C$ |
| 5.0) | $\sim Co(b > a) \wedge ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a)$ | 1, EU *y/b*; |
| 5) | $\sim Co(b > a)$ | 5.0; RCo2, PC: $(A \wedge B \wedge C) \rightarrow A$ |
| 6.1) | $\sim\sim \mathrm{GAC}(a)$ | 2, 4 + 5, PC: $[A \rightarrow (B \wedge \sim B)] \rightarrow \sim A$ |
| 6.2) | $\mathrm{GAC}(a)$ | 6.1, PC: $\sim\sim A \leftrightarrow A$ |
| 6.3) | $(\forall y) \sim Co(y > a) \wedge ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a) \wedge \mathrm{GAC}(a)$ | 1, 6.2, PC: $A, B \rightarrow A \wedge B$ |
| 6) | $(\exists x)\{(\forall y) \sim Co(y > x) \wedge ex(x) \wedge \mathrm{INE}(x)] \wedge \ldots \wedge P_{15}(x) \wedge \mathrm{GAC}(x)\}$ | 6.3, IE *a/x* |

[47] E.g. Oppenheimer and Zalta wrongly define IQM as conceivable (P. Oppenheimer, E. Zalta, *On the Logic of the Ontological Argument*, in: *Philosophical Perspectives 5: The Philosophy of Religion*, ed. J. Tomberlin, Atascadero 1991, premise 1).

This axiom affirms that, if something has all the properties shared by IQM up to being greater than any different conceivable, and is also non-conceivable, it is greater than anything having the same properties but that can be conceived. Therefore, Theor 3.2, follows as with the previous ones:

**Theor 3.2)** $(\exists x)\ \{(\forall y) \sim Co(y > a) \wedge ex(x) \wedge \mathrm{INE}(x) \wedge \ldots \wedge \mathrm{GAC}(x) \wedge \sim co(x)\}$[48]
*(Something than which a greater is not conceivable, …, is not conceivable)*

[48] Here is the detailed formal proof:

| | |
|---|---|
| 1.1) $(\exists x)\ [(\forall y) \sim Co(y > x) \wedge ex(x) \wedge \mathrm{INE}(x) \wedge \ldots \wedge P_{15}(x) \wedge \mathrm{GAC}(x)]$ | Theor 3.1. |
| 1) $(\forall y) \sim Co(y > a) \wedge ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a) \wedge \mathrm{GAC}(a)$ | 1.1, EE $x/a$ |
| 2) $co(a)$ | hp |
| 3.1) $Co[(\forall y) \sim Co(y > b) \wedge ex(b) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(b) \wedge \mathrm{GAC}(b)]$ | 1, RCo1.3 $a/b$ |
| 3.3) $Co[\sim co(b)]$ | 2, RCo1.4 |
| 3) $Co\{[(\forall y) \sim Co(y > b) \wedge ex(b) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(b) \wedge \mathrm{GAC}(b)] \wedge \sim co(b)\}$ | 3.1–3.3,RCo2,PC: $A, B \rightarrow (A \wedge B)$ |
| 4.1) $Co[ex(b) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(b) \wedge \mathrm{GAC}(b) \wedge \sim co(b)]$ | 3, RCo2, PC: $[(A \wedge B) \wedge C] \rightarrow C$ |
| 4.2) $Co[co(a)]$ | 2, RCo1.1 |
| 4.3.1) $Co[(\forall y) \sim Co(y > a) \wedge ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a) \wedge \mathrm{GAC}(a)]$ | 1, RCo1.1 |
| 4.3) $Co[ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a) \wedge \mathrm{GAC}(a) \wedge co(a) \wedge \wedge ex(b) \wedge \mathrm{INE}(b) \wedge \ldots \wedge P_{15}(b) \wedge \mathrm{GAC}(b) \wedge \sim co(b)]$ | 4.3.1 + 4.2 + 4.1, RCo2,PC: $[(A \wedge B), C] \rightarrow B \wedge C$ |
| 4.4) $[ex(a) \wedge ex(b) \wedge \ldots \wedge \mathrm{GAC}(a) \wedge \mathrm{GAC}(b) \wedge co(a) \wedge \sim co(b)] \rightarrow (b > a)$ | Ax $\mathrm{Hier}_{17}$, EU $x/a$, $y/b$ |
| 4.5) $Co\{[ex(a) \wedge ex(b) \wedge \ldots \wedge \mathrm{GAC}(a) \wedge \mathrm{GAC}(b) \wedge co(a) \wedge \sim co(b)] \rightarrow (b > a)\}$ | 4.4., RCo1.1 |
| 4) $Co(b > a)$ | 4.3, 4.5., RCo2 PC: $\{[(A \wedge B) \rightarrow C] \wedge (B \wedge A)\} \rightarrow C$ |
| 5.0) $\sim Co(b > a) \wedge ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a) \wedge \mathrm{GAC}(a)$ | 1, EU $y/b$; |
| 5) $\sim Co(b > a)$ | 5.0, PC: $(A \wedge B) \rightarrow A$ |
| 6.1) $\sim co(a)$ | 2, 4 + 5, PC: $[A \rightarrow (B \wedge \sim B)] \rightarrow \sim A$ |
| 6.3) $(\forall y) \sim Co(y > a) \wedge ex(a) \wedge \mathrm{INE}(a) \wedge \ldots \wedge P_{15}(a) \wedge \mathrm{GAC}(a) \wedge \sim co(a)$ | 1, 6.1, PC: $A, B \rightarrow A \wedge B$ |
| 6) $(\exists x)\ \{(\forall y) \sim Co(y > x) \wedge ex(\mathrm{x}) \wedge \mathrm{INE}(\mathrm{x}) \wedge \ldots \wedge P_{15}(\mathrm{x}) \wedge \mathrm{GAC}(\mathrm{x}) \wedge \sim co(\mathrm{x})\}$ | 6.3, IE $a/x$ |

| | |
|---|---|
| 1) $(\forall y) \sim Co(y > a) \wedge ex(a) \wedge \ldots \wedge \mathrm{GAC}(a)$ | Theor 3.1 |
| 2) $co(a)$ | hp |
| 3) $Co\{(\forall y) \sim Co(y > b) \wedge ex(b) \wedge \ldots \wedge \mathrm{GAC}(b) \wedge \sim co(b)\}$ | 1, RCo1.3; 2, RCo1.4; RCo2, PC: $A, B \rightarrow (A \wedge B)$ |
| 4) $Co(b > a)$ | 3, RCo2, PC: $[(A \wedge B) \wedge C] \rightarrow C$; 2, RCo1.1; 1, RCo1.1; RCo2, PC: $[(A \wedge B), C] \rightarrow B \wedge C$; Ax $\text{Hier}_{17}$ EU; RCo1.1; RCo2, PC: $\{[(A \wedge B) \rightarrow C] \wedge (B \wedge A)\} \rightarrow C$ |
| 5) $\sim Co(b > a)$ | 1, EU $y/b$, PC: $(A \wedge B) \rightarrow A$ |
| 6) $(\exists x) \{(\forall y) \sim Co(y > x) \wedge ex(x) \wedge \ldots \wedge \mathrm{GAC}(x) \wedge \sim co(x)\}$ | 2, 4 + 5, PC: $[A \rightarrow (B \wedge \sim B)] \rightarrow \sim A$; 1, PC: $A, B \rightarrow A \wedge B$; IE $a/x$ |

Theor 3.2 means that *Something than which a greater is not conceivable, exists in reality, is inconceivable as non-existent in reality, … , is greater than any different conceivable and is not conceivable*. This is really an almost incredible sequence of properties, and some authors have considered chapter 15 contradictory, because it seems to affirm that something is conceivable and is not conceivable;[49] but Anselm never affirms that IQM is conceivable![50] In chapter 15 he "only" affirms that it is possible to *conceive* that something ("*b*") than which a greater is not conceivable, …, is not conceivable: and this *is not* contradictory. This possibility is here formalized in line 3 as "*Co*{… ~*co*(*b*)}" and this proposition is not contradictory because "*Co*" and "*co*" have different meanings.

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49 J. Archambault, *Monotonic and Non-Monotonic Embeddings*, op. cit., pp. 123, 132; G. Priest, *Beyond the Limits of Thought*, Oxford 2002, p. 58.

50 M. Nowicki correctly criticizes C. Viger (*St. Anselm's Ontological Argument Succumbs to Russell's Paradox*, "International Journal for Philosophy of Religion" 2002, Vol. 52, No. 3, pp. 123–128) and underlines that in ch. 14–15 Anselm never affirms that God is conceivable (M. Nowicki, *Anselm and Russell*, "Logic and Logic Philosophy" 2006, Vol. 15, pp. 355–368, pp. 356, 361). Also G. Klima affirms that for Anselm "God is the *thought* object than which no thought object can be thought to be greater" (*Saint Anselm's Proof: A Problem of Reference, Intentional Identity and Mutual Understanding*, in: *Medieval Philosophy and Modern Times*, ed. G. Holstrom-Hintikka, Dordrecht 2000, pp. 69–87, italics added).

## 4. Conclusion

The *Proslogion* is certainly one of the deepest philosophical and theological reflections because, in its brevity, it manages to intertwine issues concerning logic, philosophy and spiritual experience.

In this regard, we are increasingly convinced that similar thought-paths can be adequately appreciated only if two different lenses are used in the analysis at the same time: that of classical philosophy and that of formal logic.

Thanks to these two analytical methods we therefore hope to have clarified that:

1) The *unum argumentum* is unique because "something than which a greater is not conceivable":

- It is a very simple notion based on a single proposition, as such differentiating from more complex ideas like the "island than which a greater island is not conceivable" (2.1.2; 3.1.2).
- It concerns only the sphere of conceivability ("*Co*"), and for this reason it can be easily accepted by everyone as an axiom: this does not happen for notions like "something greater than everything else," "something not having a greater" or the *ens perfectissimum* (2.1.3–4; 3.1.3–4, Pr GE-NG-EP).
- It involves the negation ("∼") of the conceivability of something, as such differentiating from ideas like "something conceivable as greater than everything," "something conceivable as not having a greater" and "something conceivable as not lacking any perfection," that do not involve their existence (2.1.3–4; 3.1.3–4 Pr CG-CNG-CEP).

2) The *Proslogion* has an overall unity and an ascending structure because:

- The same line of argument used in chapter 2 is applied all over the entire *Proslogion*.
- The proof in chapter 2 is only the first step of a path that begins with a prayer in which something is "proven" (footnote 24), then it ascends from the "poor" notion of IQM (2.1–3.1) to an understanding of its perfection (2.2–3.2) and transcendence (2.3–3.3) before returning more intensely to the joy of the prayer from which it started.

# APPENDIX A

Here we will prove that all steps where RCo1 and RCo2 were used indeed correspond to instances of the axiom schemata RCo1 and RCo2, that is, the propositions corresponding to A* or C (here and in the following we will leave implicit the conjunction "∧ O" as the symbol "<" will be always clearly interpreted as a strict order) in those lines are not self-contradictory.

Because of the Soundness Theorem and the Completeness Theorem,[51] this corresponds exactly to the existence of a model for the theory having as its only axiom that proposition corresponding to A* or C. It is important to stress that the model (that is, the interpretation of the constants, functions and predicates making the proposition true) might depend on the proposition, and there is no need for our purposes to provide a single interpretation that works for all propositions for which RCo1 and RCo2 are used.

Nevertheless, for the sake of brevity, we will make reference to the common structure of different proofs, corresponding to the numbering of lines, and propose an interpretation of the symbols involved that is as uniform as possible. Since:

- a sentence and its double negation (3.2 and 3.3) are equisatisfiable;
- if the conjuction of multiple sentences (3 or 4.3) is satisfiable, then each conjunct (3.1, 3.3 and 4.1, or 4.2 respectively) also is;
- a sentence (3.1) and another that is identical to it, except for the fact that one constant has been uniformely replaced by another (4.3.1), are equisatisfiable;
- if the consequent (4) in a hypothetical is satisfiable, then the hypothetical also is (4.5),

we can reduce the problem to considering lines 3, 4.3 and 4 in each proof.

Consider now a structure that has as domain M = {0,1}^16, that is, the sixteenth cartesian power of a set with 0 and 1 as its only elements; in other words the set of all possible sequences of zeros and ones of length sixteen.

Now interpret predicates $ex(x)$, INE($x$), $P_3(x)$, … , $P_{15}(x)$, $co(x)$ the following way:

- the $n$-th predicate in the list corresponds to the sequence having the $n$-th coordinate equal to 1;
- except for $co(x)$, which corresponds to the last coordinate being equal to 0.

[51] L. Henkin, *The Completeness of the First-Order Functional Calculus*, op. cit.

Interpret moreover "$y > x$" and "$Co(y > x)$" as the lexicographic order,[52] meaning that one sequence "$s$" is greater than another "$q$" if and only if "$s$" takes value 1 and "$q$" takes value 0 on the first position where they disagree. For instance, the lexicographic order on {0,1}^3 is:

$$(0, 0, 0) < (0, 0, 1) < (0, 1, 0) < (0, 1, 1) < (1, 0, 0) < (1, 0, 1) < (1, 1, 0) < (1, 1, 1)$$

Finally, interpret:

- "$b$" as (1, … , 1) in all relevant lines of all proofs;
- and interpret "$a$" as (0, … , 0) up to the proof in footnote 41 included and then as (1, 0, … , 0), then (1, 1, 0, …, 0), then (1, 1, 1, 0, … , 0), and so on where the first zero is in $n$-th position in the proof that IQM has the n-th predicate (here the sixteenth predicate would be "being *not* conceivable"). So, for instance, in the proof in footnote 48, "$a$" will be interpreted as (1, … , 1, 0).

Then one just needs to check that this interpretation works. We are going to present in detail as examples only the case of the proof of Theor 1.1 and that of Theor 3.2. As already explained, it is enough to check that the propositions inside "*Co*" in 3, 4.3 and 4. are satisfied.

First the case of Theor 1.1:

- For line 3, note that "$b$," being interpreted as (1, … , 1) is indeed greater than any sequence in the lexicographic order, which is the interpretation of the "$Co(y > x)$" relation, so that "$(\forall y) \sim Co(y > b)$" is satisfied; and its first coordinate is 1, making it fall within the extension of the predicate "ex."
- For line 4.3, note that (0, … , 0), which is our interpretation of "$a$," has 0 as its first coordinate, while (1, … , 1), which is our interpretation of "$b$," has 1.
- For line 4, note that (1, … , 1) is greater than (0, … , 0) in the lexicographic order.

Now for Theor 3.2:

[52] The 2-ary predicate "$Co(x < y \wedge P(x) \wedge P(y))$" of the proof in footnote 38 will be interpreted in the same way. Note that there is no need here to fix an interpretation of the predicate "P" itself since in our setting "Co" forms a new predicate of the signature when applied to any formula with some free variables. On the other hand, in line 4.3 of the proof in footnote 37 we have to abandon this interpretation of "$Co(x < y)$" and we can interpret it, e.g., as the empty relation, that is, the one that never holds no matter which couple of elements we consider.

- For line 3, "$(\forall y) \sim Co(y > b)$" is satisfied for the same reason as above, and all unary predicates hold for "b" because all coordinates of the interpretation of "$b$" are equal to 1. Remember that:
- "*co*" corresponds to the last element in the sequence being equal to 0, so that indeed "$\sim co(b)$" is satisfied.
- Finally, "GAC($b$)" holds since under this interpretation "$b$" is greater with respect to "<" than all other elements of the domain and therefore greater than all elements with the last coordinate equal to 0.
- For line 4.3, the part about "$b$" is satisfied because of what we have already said, and the part about "$a$" is satisfied because its interpretation, namely, $(1, \dots, 1, 0)$, also has the right values on each position with respect to the unary predicates. Moreover, $(1, \dots, 1, 0)$ is precisely the greatest of all sequences that end with 0 (that is, those in the extension of "$co(x)$") in the lexicographic order, so that "GAC($a$)" is satisfied.
- For line 4, we have exactly the same as for line 4 of the proof of Theorem 1.1.

The reader is invited to check on their own similar cases of the subsequent proofs. Note that even though "$Co(x < y)$" and "INE($x$)" are defined in terms of other predicates, for the purpose of this section there is no need of their interpretation being related in any way to that of "<" and "*ex*" respectively, since any law governing the relation between the two will arise from other axioms in our theory, and not from the propositional calculus inference rules themselves.

## APPENDIX B

In this appendix we will prove some interesting features of our theory (call it TC) that may allow one to better understand the contents of the *Proslogion*.

Let us begin by proving that, if we further allow for conceiving the exact negation of a proposition one has understood, then the theory becomes inconsistent. More formally, we say that if one adds to the axiom schema RCo1 the following case:

RCo1.2: $P(a) \rightarrow Co[\sim P(a)]$
*(For example, "It is conceivable that Socrates does not run")*

then every consistent proposition is conceivable (Lemma B.1) and as a consequence the theory becomes inconsistent[53] (Theorem B.2). Notice that this would also be inconsistent with IQM being inconceivable as non-existent, since we prove that it exists in reality (see Theor 2.1). We call TC.2 = TC + RCo1.2

**Lemma B.1)** *In TC.2, that which is consistent is conceivable, that is:*

Let $A$ be such that A $\nvdash \bot$. Then $TC.2 \vdash Co(A)$.

*Proof.*
Let $B$ be non-tautological and such that $TC.2 \vdash B$ (for instance, $(\forall y) \sim Co(y > a)$). Then:

$$Co(\sim B)$$

holds by RCo1.2 because $\sim B$ is not a contradiction, and since $TC.2 \vdash (\sim A \rightarrow B)$, by RCo1.1. we have:

$$TC.2 \vdash Co(\sim A \rightarrow B)$$

We conclude by RCo2, Co($\sim B$) and *modus tollens* (PC: $[(\sim A \rightarrow B) \wedge \sim B] \rightarrow A$) along with the assumption that $A$ is consistent, that:

$$TC.2 \vdash Co(A).$$

**Theorem B.2)** *TC.2 is inconsistent.*

*Proof.*
From Pr IQM we can deduce $\sim Co(b > a)$, that is, TC.2 $\vdash \sim Co(b > a)$, but clearly $(b > a)$ is consistent, so that it should be $TC.2 \vdash Co(b > a)$.

As an alternative proof we can notice that $\forall x \exists y\, (y > x)$ is consistent (it is satisfied, for example, in the structure of the natural numbers), so that:

$$TC.2 \vdash Co[\forall x \exists y\, (y > x)]$$

but we will prove (Lemma B.4) that

$$TC \vdash Co[\forall x \exists y\, (y > x)]$$

[53] Reasoning along these same lines one can see that in the analysis of the consequences of Pr GE in 3.1.3, propositions 4) and 5) do indeed contradict each other, since we conceive the negation of another proposition we have proved. The same does not hold, e.g., in the case of Pr CG.

The following lemmas describe the way conceivability relates to quantifiers and an application thereof.

**Lemma B.3)** *In TC quantifiers come out of "Co"*[54]

Let $Q_1, \ldots, Q_n$ be quantifiers, $P$ an $n$-ary predicate and let $TC$ be our theory.

Then $TC \vdash Co[Q_1x_1 \ldots Q_nx_n P(x_1, \ldots, x_n)] \rightarrow Q_1x_1 \ldots Q_nx_n Co[(P(x_1, \ldots, x_n)]$

*Proof.*
Let $Q_i$ be a quantifier. We denote by $Q^-_i$ the opposite quantifier, that is, if $Q_i$ is a particular quantifier then $Q^-_i$ is a universal quantifier and vice versa. Let us prove the result by induction on the number of quantifiers. We want to prove that:

$$TC, Co[Q_1x_1 \ldots Q_nx_n P(x_1, \ldots, x_n)], {\sim}Q_1x_1 \ldots Q_nx_n Co[(P(x_1, \ldots, x_n)] \vdash \bot$$

Consider then:

$$Co[Q_1x_1 \ldots Q_nx_n P(x_1, \ldots, x_n)] \text{ and } Q^-_1 x_1 \ldots Q^-_nx_n {\sim}Co[(P(x_1, \ldots, x_n)]$$

One among $Q_1$ and $Q^-_1$ is a particular quantifier, so that we can replace its variable by a corresponding Henkin constant and then eliminate the other, which is, the universal quantifier, by substituting with the same constant. We get:

$$Co[Q_2x_2 \ldots Q_nx_n P(c, x_2, \ldots, x_n)]$$

and

$$Q^-_2x_2 \ldots Q^-_nx_n {\sim}Co[(P(c, x_2, \ldots, x_n)]$$

We can thus see $P(c, x_2, \ldots, x_n)$ as an $(n\text{-}1)$-ary predicate and apply the inductive hypothesis to see that these two contradict one another.

[54] We consider this lemma a feature that any theory of conceivability should have, rather than a logical defect in Anselm's assumptions as we have interpreted them. On the other hand, it seems that the universal quantifier in front of "Co" should not pass into it: from the fact that for each team it is conceivable that it will win the championship it does not follow that it is conceivable that all of them (at the same time) will win.

**Lemma B.4)** *As a consequence of this lemma we have that:*

$$TC \vdash \sim Co[\forall x \exists y\ (y > x)].$$

for if we assume by contradiction

$$Co[\forall x \exists y\ (y > x)]$$

we can deduce:

$$\forall x \exists y\ \mathrm{Co}(y > x)$$

and from it

$$\sim \exists x \forall y \sim \mathrm{Co}(y > x),$$

which contradicts Pr IQM, so that the assumption must be denied:

$$\sim Co[\forall x \exists y\ (y > x)].$$

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## Summary

This article proposes an interpretation of St Anselm's *Proslogion* that highlights its overall structure and theoretical core. The analysis is conducted in two stages: (a) discussion of the text and its previous interpretations in order to clarify Anselm's premises and reasoning; (b) formal analysis of the arguments through symbolic logic, and comparison with other ontological arguments. More precisely, we describe a first-order theory corresponding to our interpretation of Anselm's commitments and show that his conclusions follow from these axioms. The theses that this study will defend are the following: (a) the *unum argumentum* applies only to "id quo maius cogitari nequit" and not to other similar concepts, such as that of "most perfect being"; (b) the treatise has an overall unity that has an ascending trend; (c) our original formalization of the *unum argumentum* not only captures the essence of the *Proslogion*, but also clarifies some features of conceivability.


**Key words:** *Proslogion*, St Anselm, symbolic logic, conceivability, natural theology


## Streszczenie

### *Proslogion*: filozofia i logika

Artykuł ten przedstawia interpretację *Proslogionu* św. Anzelma, która ukazuje jego strukturę oraz teoretyczny rdzeń. Analiza została przeprowadzona w dwóch etapach: (1) dyskusja dotycząca tekstu i jego dotychczasowych interpretacji, służąca rozjaśnieniu przesłanek, na których bazuje Anzelm, oraz jego rozumowania; (b) formalna analiza argumentacji z użyciem logiki symbolicznej i porównanie jej z innymi dowodami ontologicznymi. Precyzyjnie rzecz ujmując, opisujemy teorię pierwszego rzędu odpowiadającą naszej interpretacji założeń Anzelma i pokazujemy, że jego wnioski wynikają z owych aksjomatów. Niniejsze studium broni następujących tez: (1) *unum argumentum* ma zastosowanie tylko do „id quo maius cogitari nequit", a nie do innych podobnych pojęć, takich jak „byt najdoskonalszy"; (b) traktat jest spójny i charakteryzuje się jednością z tendencją wzrastającą; (c) nasza oryginalna formalizacja *unum argumentum* nie tylko uchwytuje istotę *Proslogionu*, lecz także rozjaśnia pewne cechy pojmowalności.


**Słowa kluczowe:** *Proslogion*, św. Anzelm, logika symboliczna, pojmowalność, teologia naturalna